Forum for Electromagnetic Research Methods and Application Technologies (FERMAT)

Transformation Electrostatics

By:
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Abstract: Transformation electrostatics is a d.c. reduction of the transformation electromagnetic or transformation optics, which provides a freedom way to control the electric currents and potentials. Transformation optics/electromagnetics has been used to design and create a lot of novel devices theoretically and numerically, but few of them have been realized due to the difficulties in fabrications of inhomogeneous and anisotropic permittivity and permeability. However, transformation statics provides us an efficient methodology to design and realize novel d.c. devices freely. Using the analogy between electrically conducting materials and resistor networks, such d.c. devices can be designed using the circuit theory. We have fabricated a series of d.c. invisibility cloaks, exterior cloaks, active cloaks, concentrators, carpet cloaks, and illusion devices, etc. The measurement results agree exceptionally well with theoretical predictions and simulation results, showing perfect performance. Manipulation of steady currents with the control of anisotropic conductivities has a lot of potential applications, such as electric impedance tomography, graphene, natural resource exploration, and military hiding.

Keywords: Transformation electrostatics, conducting medium, resistor network, dc cloak, dc transformation devices, control of dc current.

IET Seminar on Transformation Optics. 26 October 2012

References

1. F. Yang, Z. L. Mei, T. Y. Jin, and T. J. Cui, "D. C. electric invisibility cloak," *Physical Review Letters*, vol.109, 053902 (2012)
2. W. X. Jiang, C. Luo, Z. L. Mei, and T. J. Cui, "An ultrathin but nearly perfect dc electric cloak," *Applied Physics Letters*, vol. 102, 014102 (2013)
3. M. Liu, Z. L. Mei, X. Ma, and T. J. Cui, "D. C. illusion and its experimental verification," *Applied Physics Letters*, vol. 101, 051905 (2012).
4. Z. L. Mei, Y. S. Liu, F. Yang, and T. J. Cui, "D. C. carpet cloak," *Optics Express*, vol. 20, 25758-25765 (2012)
5. W. X. Jiang, C. Y. Luo, and T. J. Cui, "Enhancement of current density by dc electric concentrator," *Scientific Reports*, vol. 2, 956; DOI:10.1038/srep00956 (2012)

References

6. F. Yang, Z. L. Mei, X. Y. Yang, T. Y. Jin, and T. J. Cui, "Negative conductivity material makes dc invisibility cloak hide object at a distance," *Advanced Functional Materials*, DOI: 10.1002/adfm.201300226 (2013)
7. Q. Ma, Z. L. Mei, S. K. Zhu, T. Y. Jin, and T. J. Cui, "Experiments on active cloaking and illusion for Laplace equation," *Physical Review Letters*, vol. 111, 173901 (2013)

Contents

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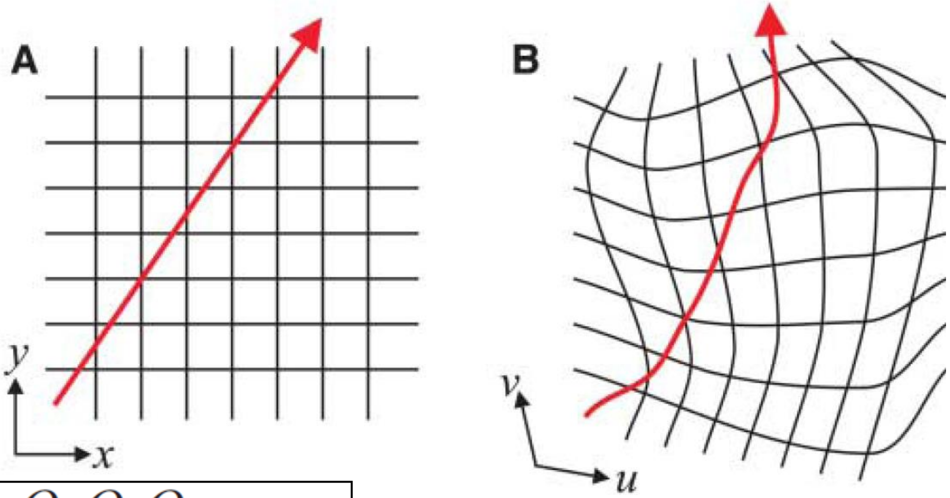
◆ **Motivation of This Work**

◆ **Transformation Electrostatics Devices**

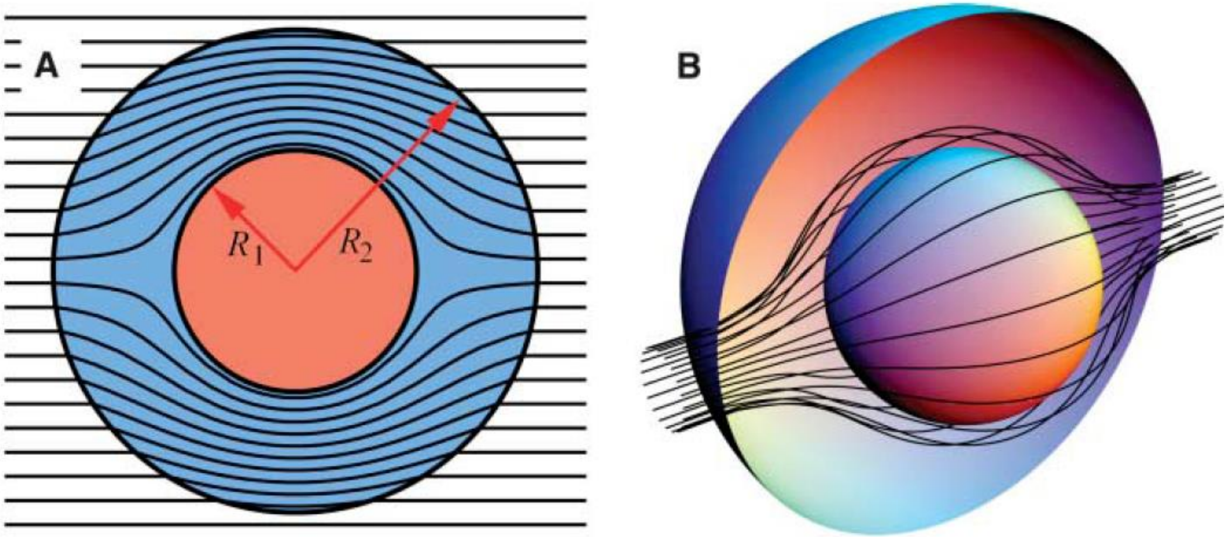
◆ **Conclusions**

Transformation Optics



$$\varepsilon'_u = \varepsilon_u \frac{Q_u Q_v Q_w}{Q_u^2},$$

$$\mu'_u = \mu_u \frac{Q_u Q_v Q_w}{Q_u^2}, \text{ etc.}$$



- ◆ **Elegant**
- ◆ **General**
- ◆ **Accurate**

Problems

- Comparing with the large amounts of theoretical work, only limited experiments have been conducted for transformation optics.

dc Transformation Optics

- In the past a few years, the dc electric cloaks and magnetic cloaks have been theoretically proposed using the non-uniqueness property of anisotropic conducting materials and optical transformation.

Greenleaf et al., *Physiol. Meas.* **24**, 413 (2003);
Wood & Pendry, *J. Phys. Condens. Matter* **19**, 076208 (2007).

dc Magnetic Cloaks

Two Experiments Based on Superconducting Materials:

- Narayana, S. & Sato, Y. DC magnetic cloak. *Adv. Mater*. **24**, 71 (2012)

Following the idea by Wood & Pendry

Experimental Realization of a Magnetic Cloak

Fedor Gömöry,[1] Mykola Solovyov,[1] Ján Šouc,[1] Carles Navau,[2] Jordi Prat-Camps,[2] Alvaro Sanchez[2]*

23 MARCH 2012 VOL 335 **SCIENCE**

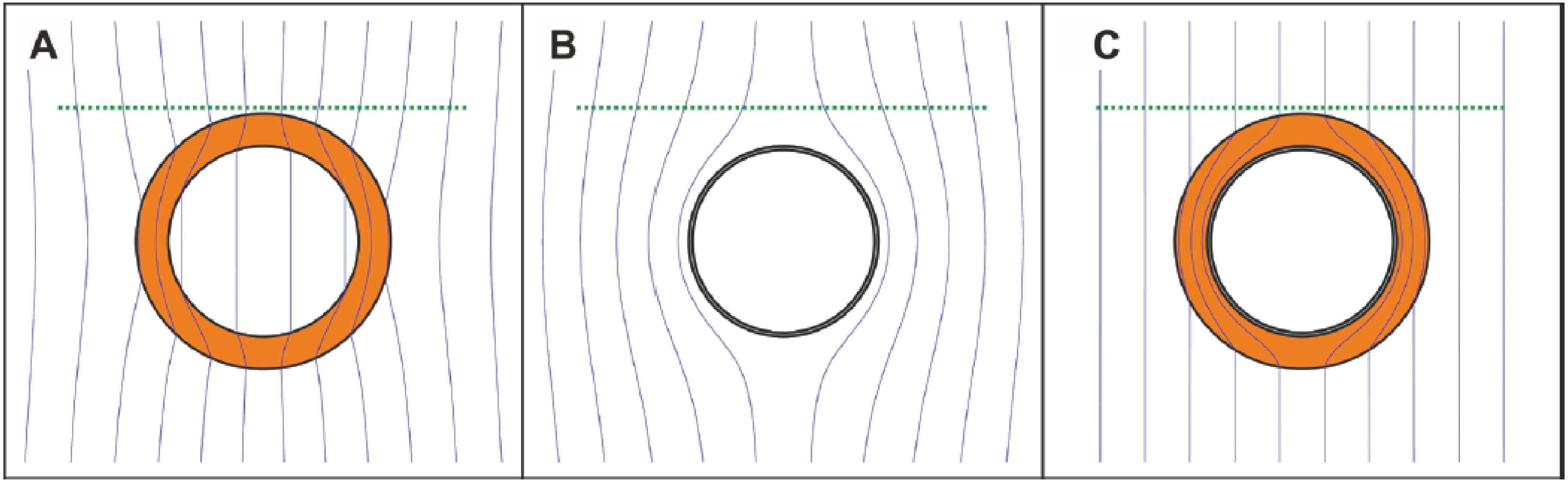


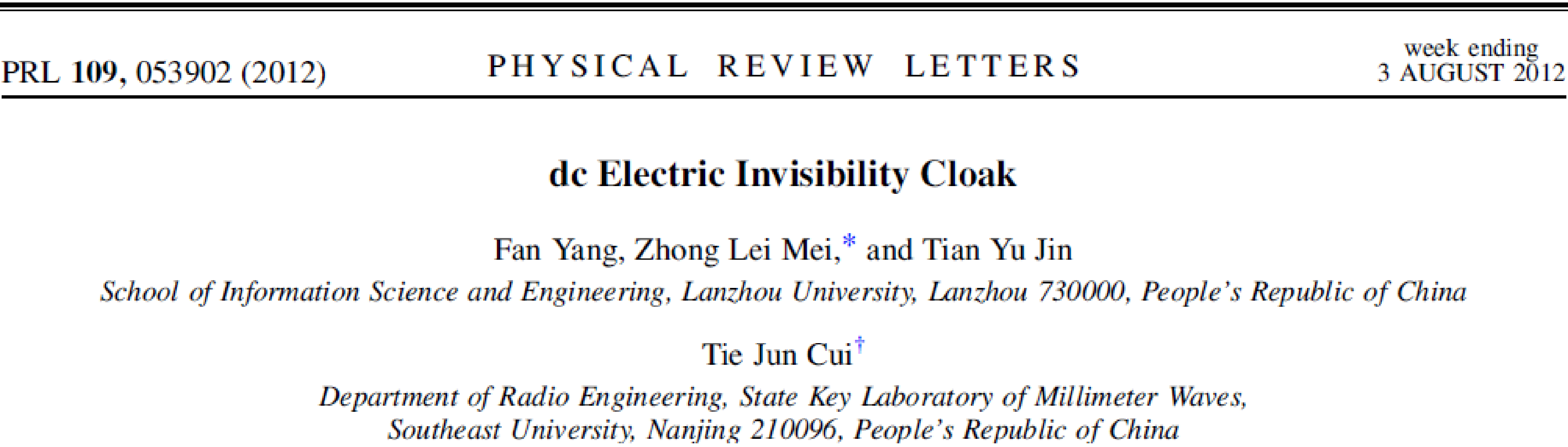
Fig. 1. Calculated field lines for (**A**) a single cylindrical magnetic shell with $\mu = 3.54$, attracting fields and having some field penetration in its interior; (**B**) a single cylindrical superconducting shell with $\mu = 0$ repelling field lines; and (**C**) a cylindrical bilayer with an inner superconducting layer ($\mu = 0$) of interior (exterior) radius of $R_0 = 0.96\ R_1$ (R_1) and an outer magnetic layer with $R_2/R_1 = 1.34$ with $\mu_2 = 3.54$, fulfilling Eq. 1. These values are chosen to approximate those used in the experiments. Green dotted lines denote the measuring lines in the experiments.

Our Motivation

To propose a general method to design and realize perfectly d.c. electric cloak;

To propose, design, and realize a series of transformation electrostatic devices.

Perfect dc Electric Cloak



PRL **109,** 053902 (2012) PHYSICAL REVIEW LETTERS week ending 3 AUGUST 2012

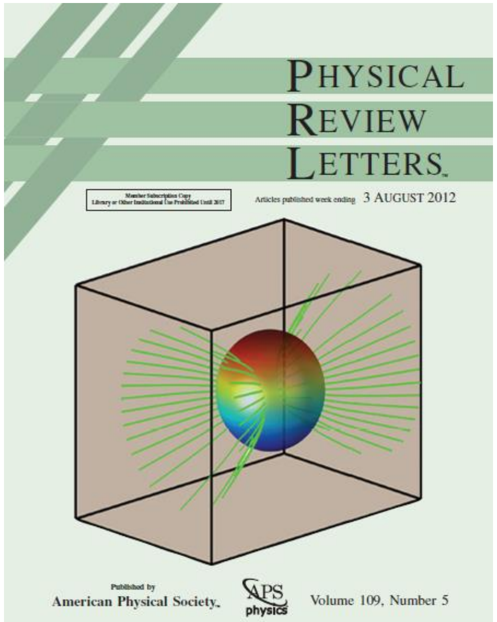
dc Electric Invisibility Cloak

Fan Yang, Zhong Lei Mei,[*] and Tian Yu Jin
School of Information Science and Engineering, Lanzhou University, Lanzhou 730000, People's Republic of China

Tie Jun Cui[†]
Department of Radio Engineering, State Key Laboratory of Millimeter Waves,
Southeast University, Nanjing 210096, People's Republic of China

A much cheaper cloak with perfect cloaking effect

Yang, et al., *Physical Review Letters,* vol.109, 053902 (2012)


PHYSICAL
REVIEW
LETTERS
Articles published week ending 3 AUGUST 2012
Published by
American Physical Society
APS physics
Volume 109, Number 5

Transformation Electrostatics

Theoretical Background

- **Fundamental equations for steady electric fields**

$$\nabla \cdot \mathbf{J} = 0 \qquad \nabla \times \mathbf{E} = 0 \qquad \mathbf{J} = \overleftrightarrow{\sigma} \cdot \mathbf{E}$$

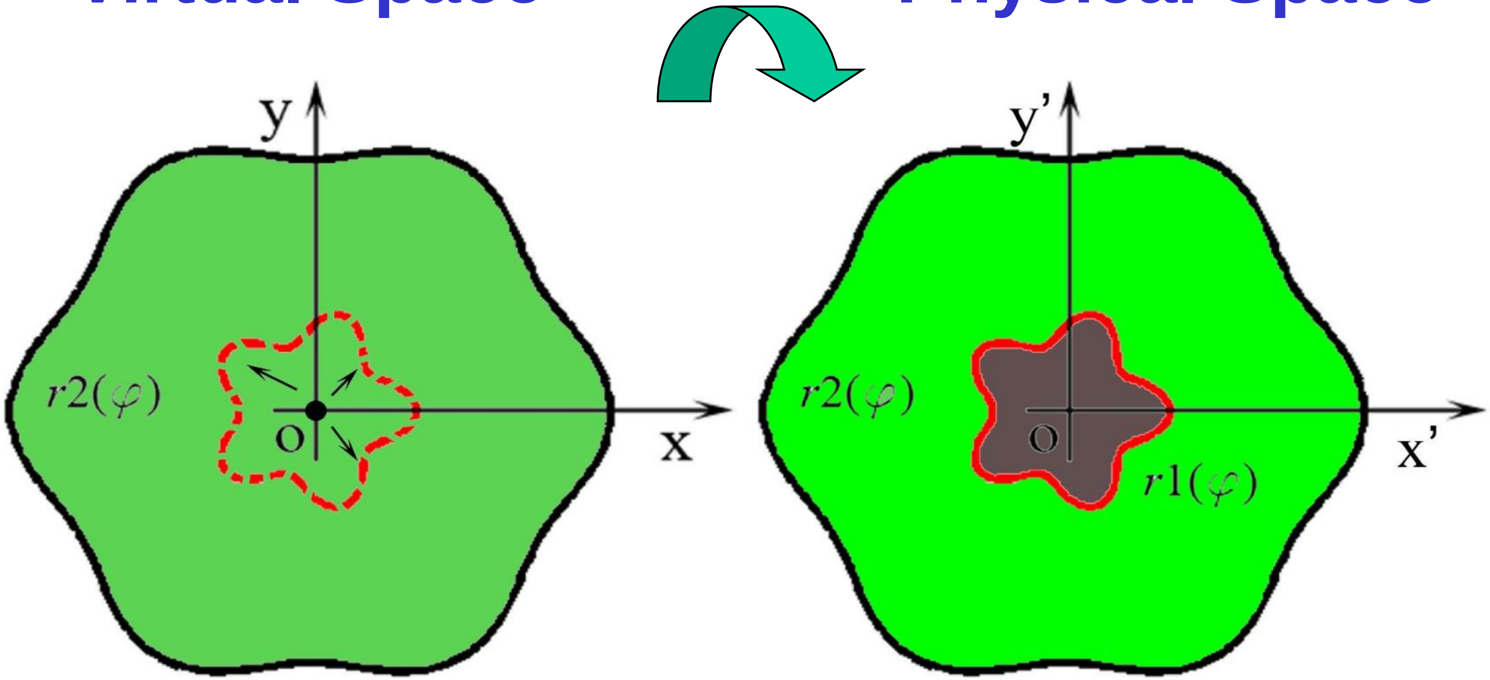
- **These equations look similar to those for electrostatics**

$$\nabla \cdot \mathbf{D} = 0 \qquad \nabla \times \mathbf{E} = 0 \qquad \mathbf{D} = \overleftrightarrow{\varepsilon} \cdot \mathbf{E}$$

- **They are form invariant under general coordinate transformation. Hence the transformation optics is valid.**

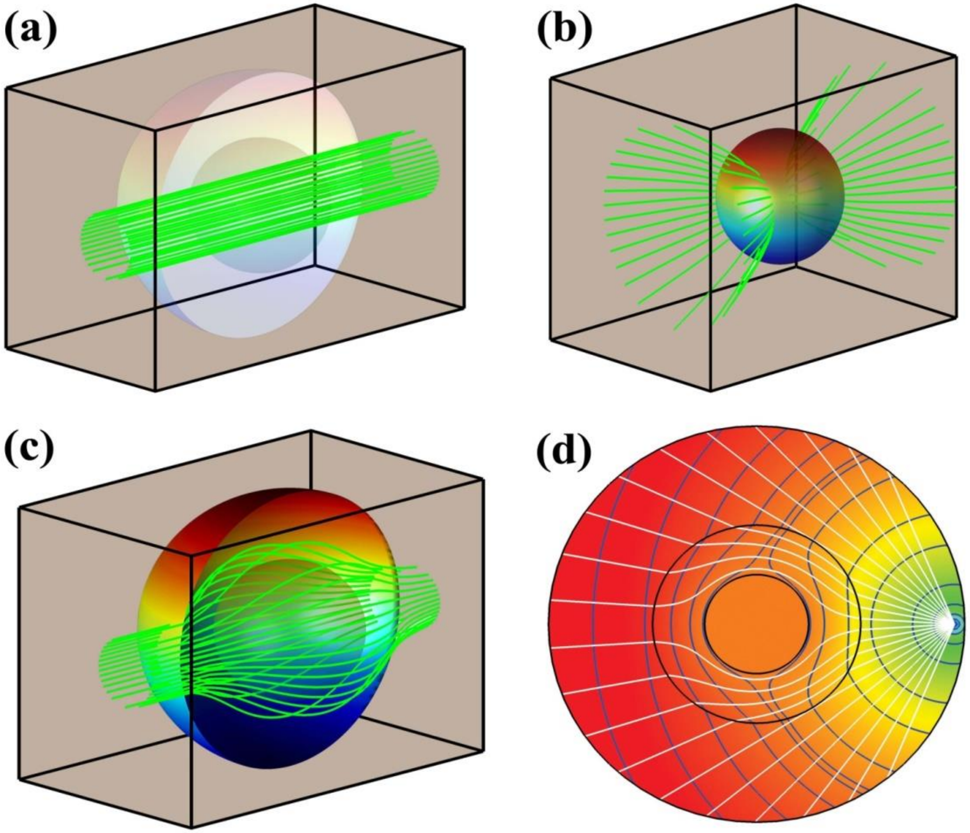
Transformation Electrostatics

Virtual Space **Physical Space**



$$\overleftrightarrow{\sigma} \Longrightarrow \overleftrightarrow{\sigma}' = \frac{\mathbf{A}\cdot\overleftrightarrow{\sigma}\cdot\mathbf{A}^{\mathrm{T}}}{\det(\mathbf{A})}$$

dc Electric Cloak



(a) No objects;

(b) Uncovered object;

(c) Covered object;

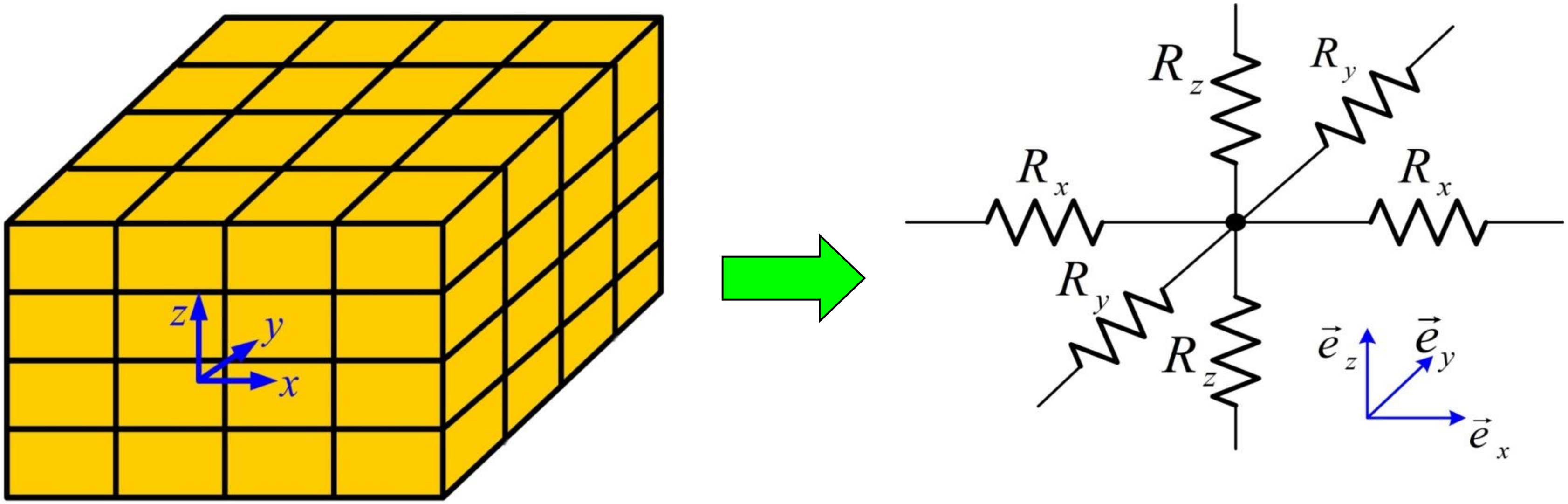
(d) 2D case.

The conductivity profile contains zero point and singular value.

$$\rho' = f(\rho) = \frac{b-a}{b}\rho + a, \quad \varphi' = \varphi, \quad z' = z,$$

$$\bar{\bar{\sigma}}' = \Lambda\left[\frac{\rho'-a}{\rho'}, \frac{\rho'}{\rho'-a}, \frac{\rho'-a}{\rho'}\left(\frac{b}{b-a}\right)^2\right]\bar{\bar{\sigma}}$$

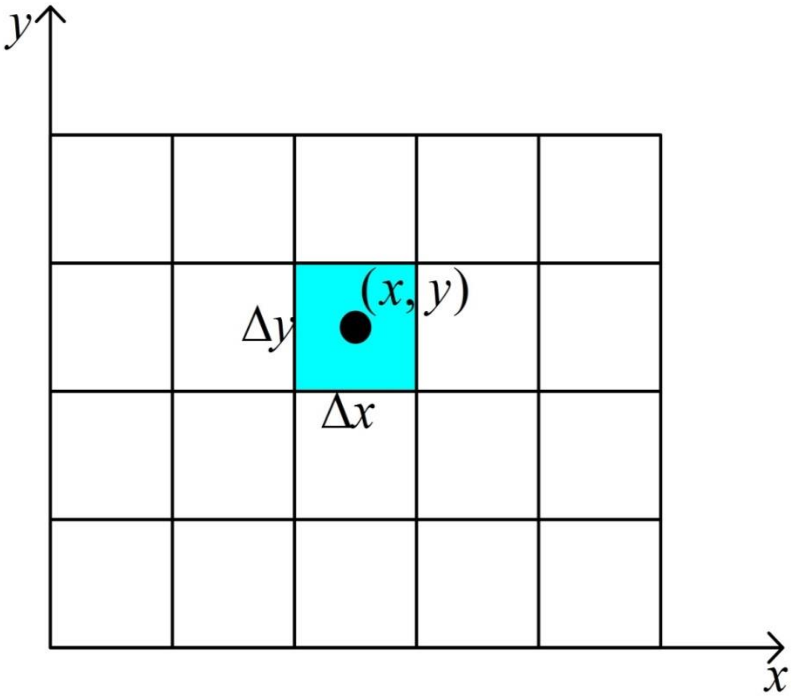
Emulation of Conductivity Tensor with Resistor Network

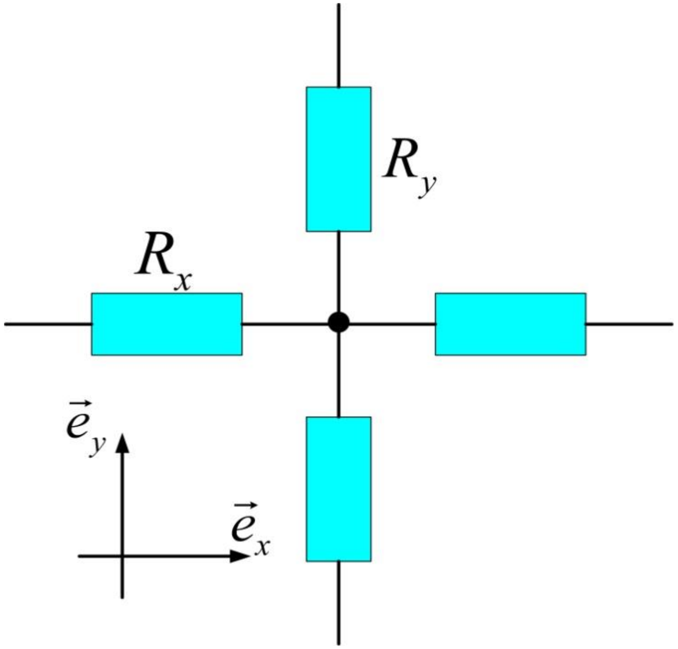

z
y
x
R z
R y
R x
R x
R y
R z
e⃗ z
e⃗ y
e⃗ x

$$R_x = \frac{L}{\sigma_x \cdot S} = \frac{dx}{\sigma_x \cdot dy \cdot dz}$$

A bulk conducting medium **A periodic resistor network**

2D: Rectangular Grids

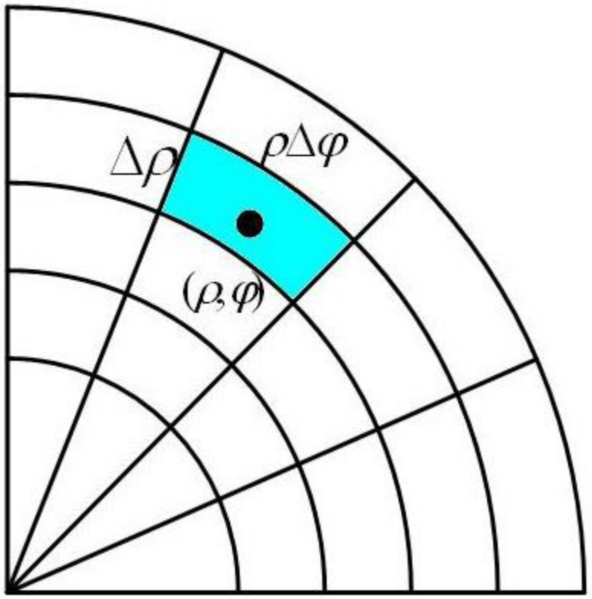



R_y
R_x
$\vec{e}_y$
$\vec{e}_x$

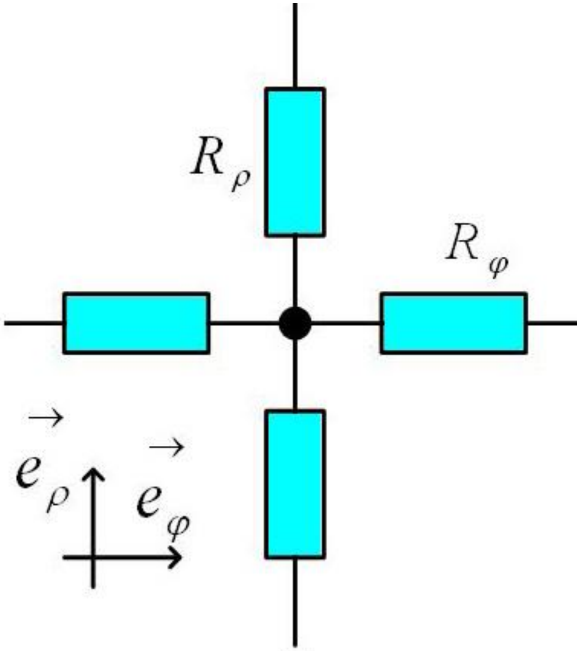
$$R_x = \frac{L}{\sigma_x \cdot S} = \frac{dx}{\sigma_x \cdot dy \cdot h}$$

$$R_y = \frac{L}{\sigma_y \cdot S} = \frac{dy}{\sigma_y \cdot dx \cdot h}$$

2D: Cylindrical Grids



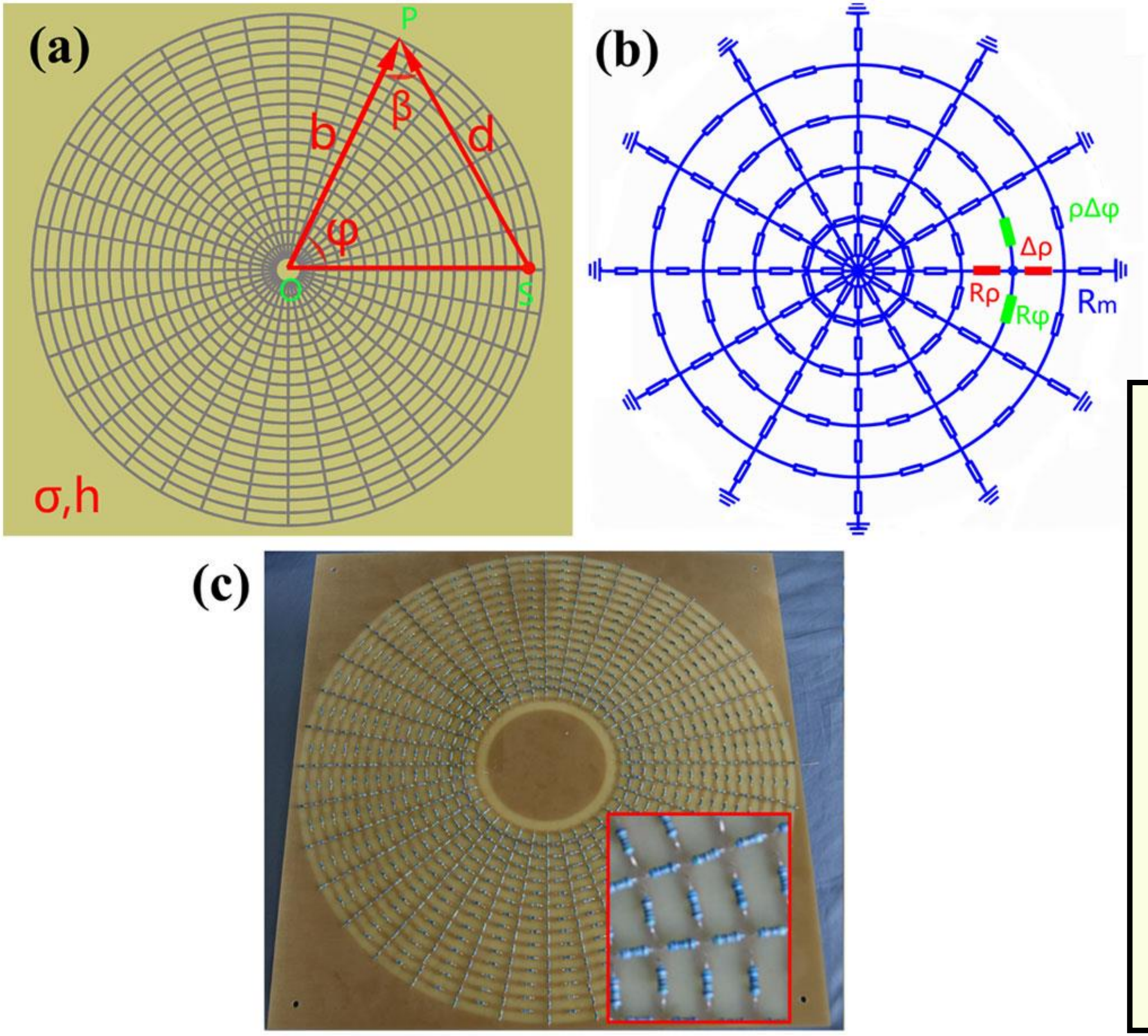
(a)


R_ρ
R_φ
$\vec{e}_\rho$
$\vec{e}_\varphi$

(b)

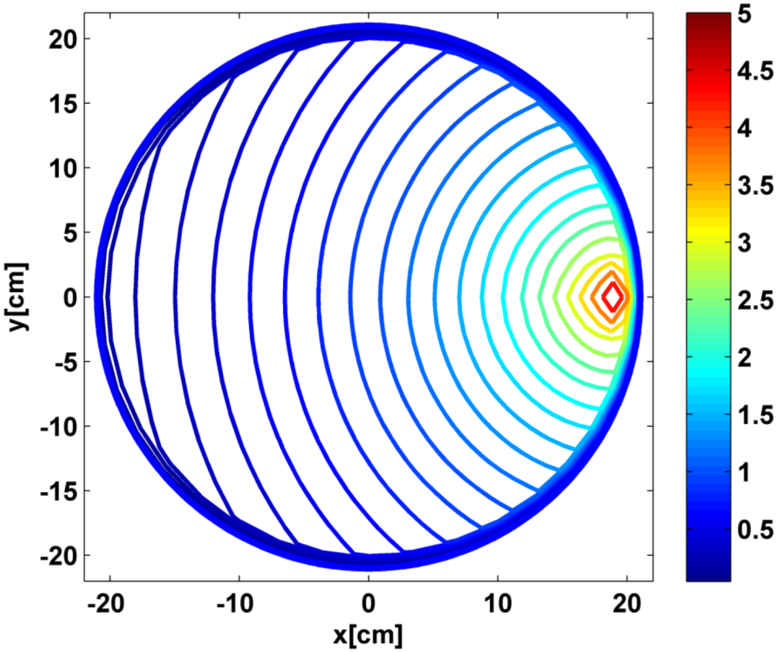
$$R_\rho = \frac{\Delta\rho}{\sigma\rho\Delta\varphi h} \qquad R_\varphi = \frac{\rho\Delta\varphi}{\sigma\Delta\rho h}$$

dc Electric Cloak


(a)
P
β
b
d
φ
O
S
σ,h
(b)
ρΔφ
Δρ
Rρ
Rφ
Rm
(c)

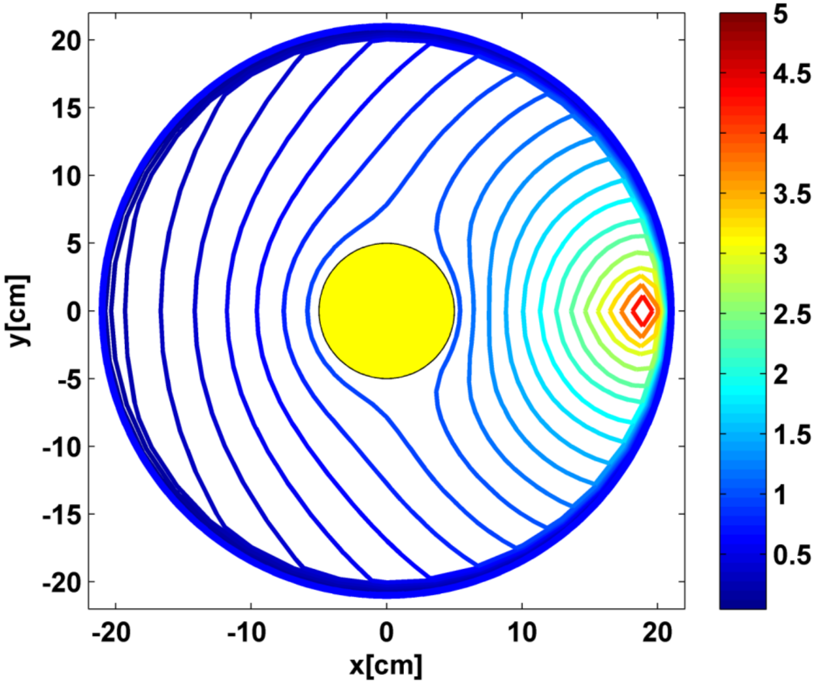
In the dc case, both zero point and singular value of conductivity have real physics meaning, and hence perfect cloaking effect can be realized.

Cloaking Performance

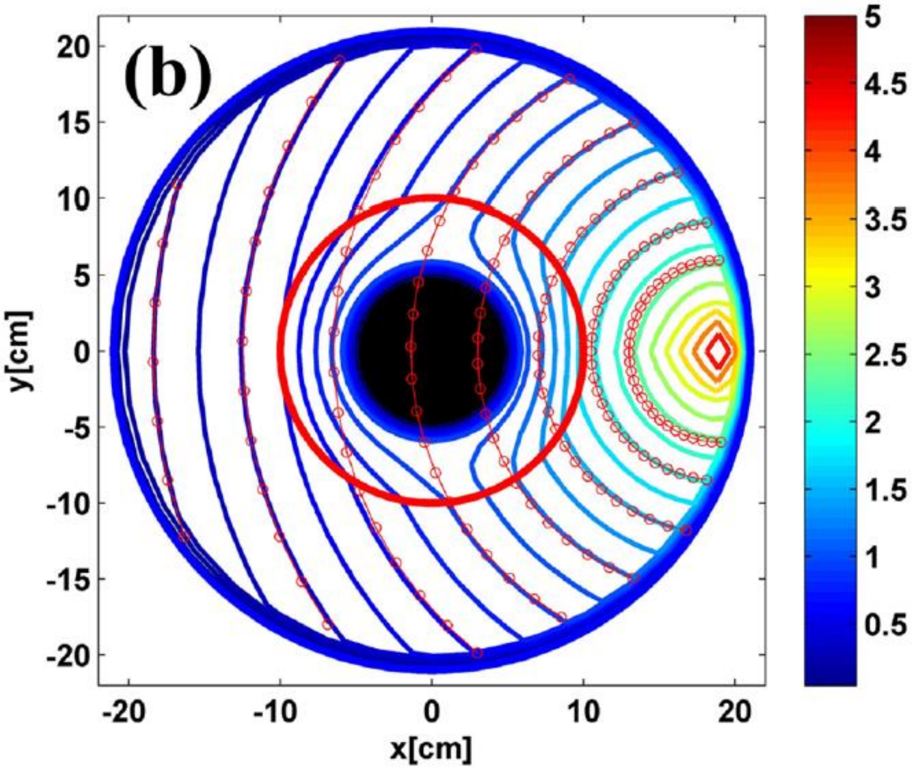


The isotropic and homogeneous background material.

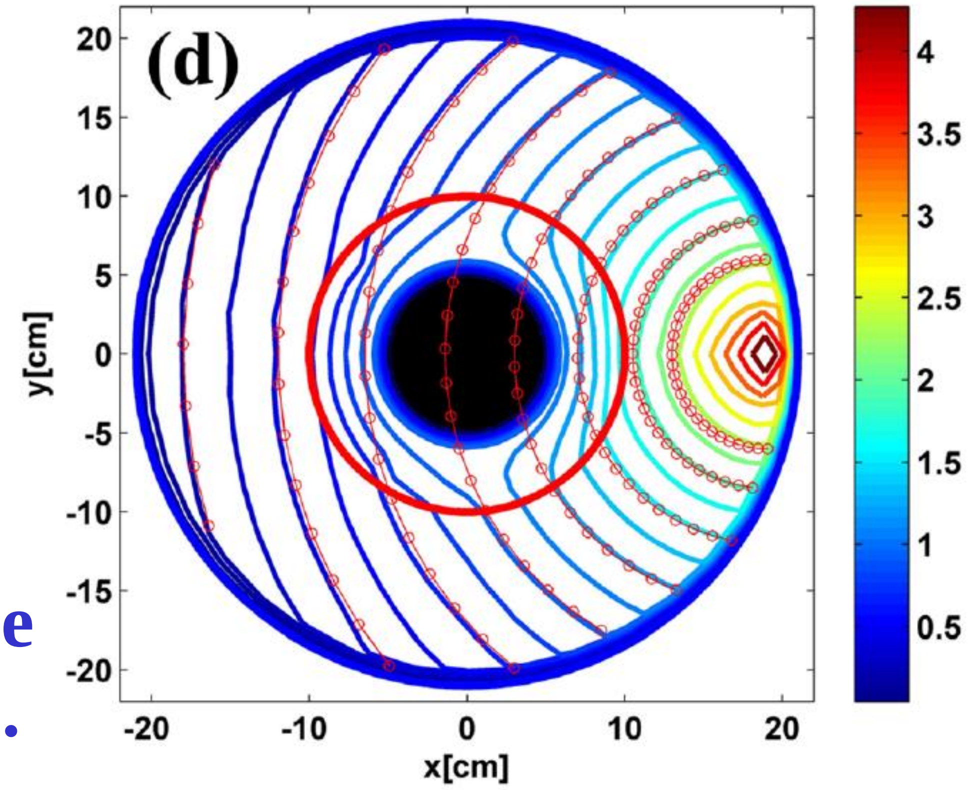
The central area is a perfect conductor.



Cloaking Performance

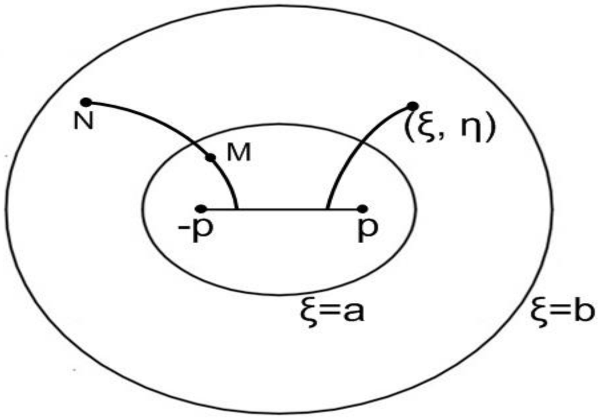


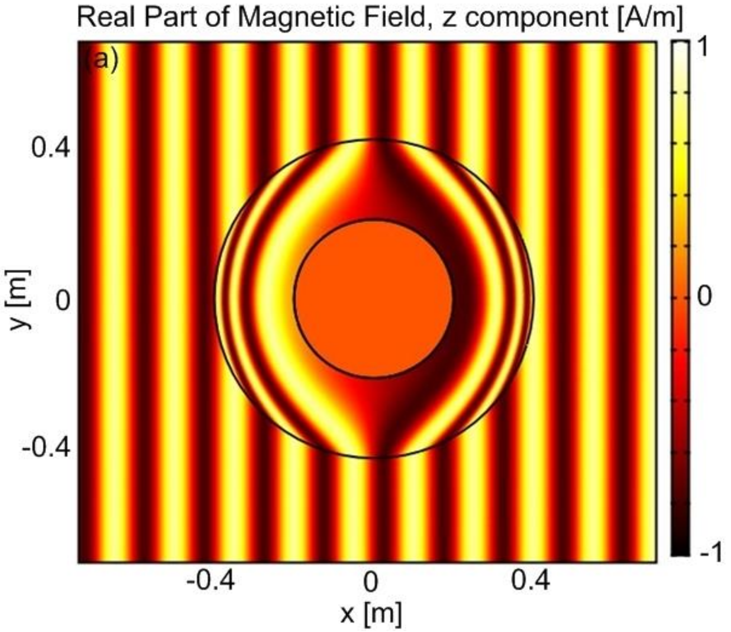
The central region is cloaked.

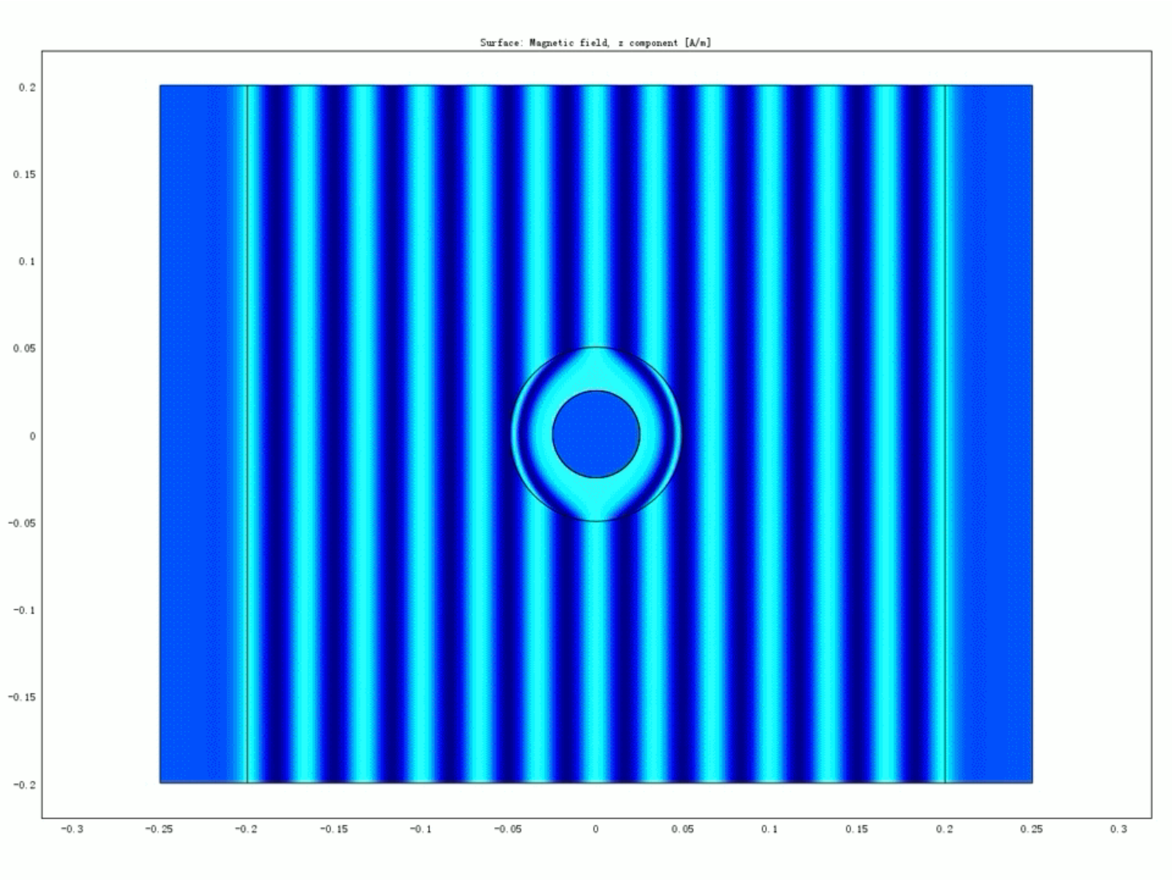


Perfect cloaking performance is observed in measurements.

Nearly Perfect Cloak without Singularity

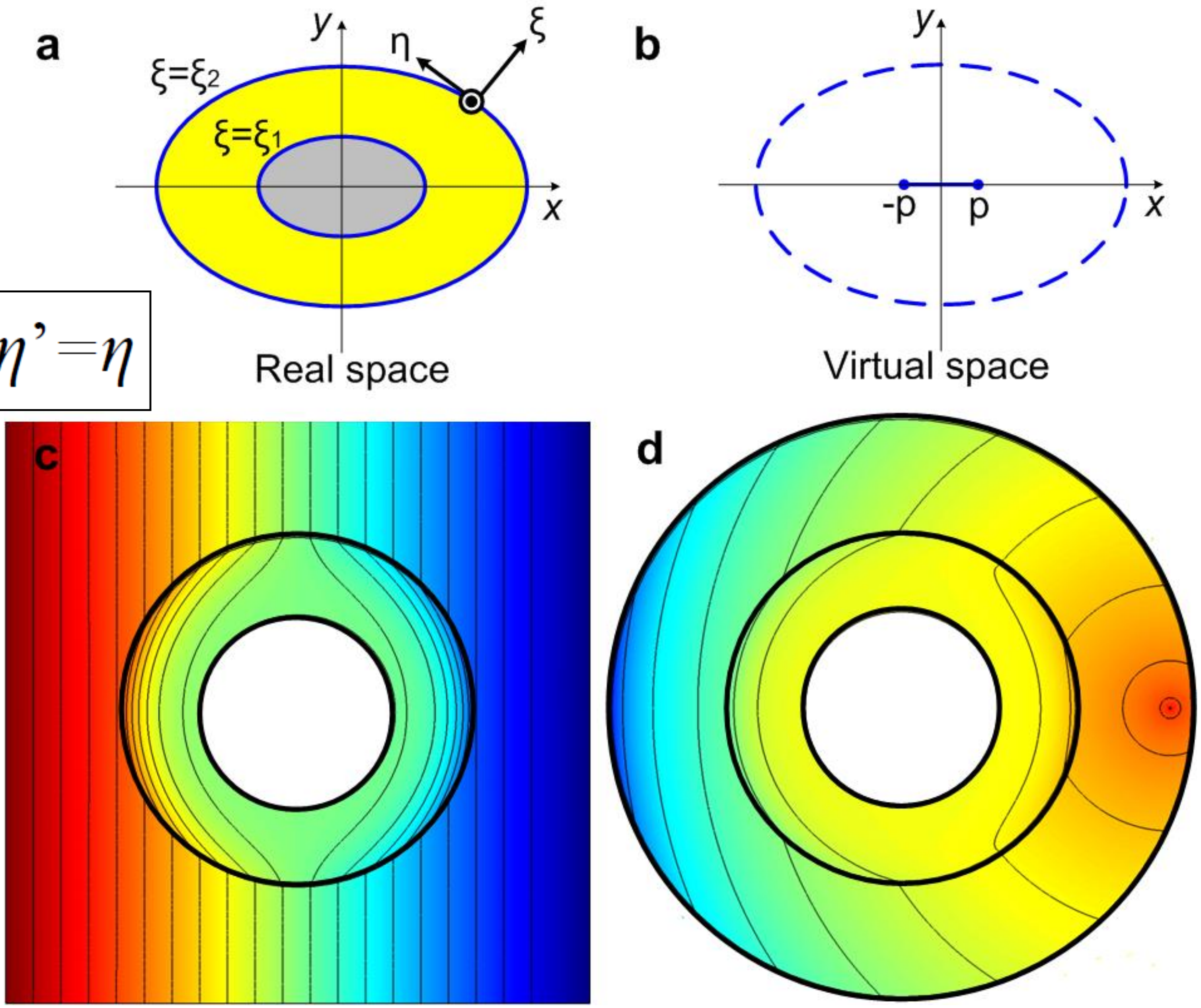

N
M
(ξ, η)
-p
p
ξ=a
ξ=b





Jiang, et al., *APL* 93, 194102 (2008)

Further Work on dc Cloak


a
y
η
ξ
ξ=ξ2
ξ=ξ1
x
Real space
b
y
-p
p
x
Virtual space
c
d

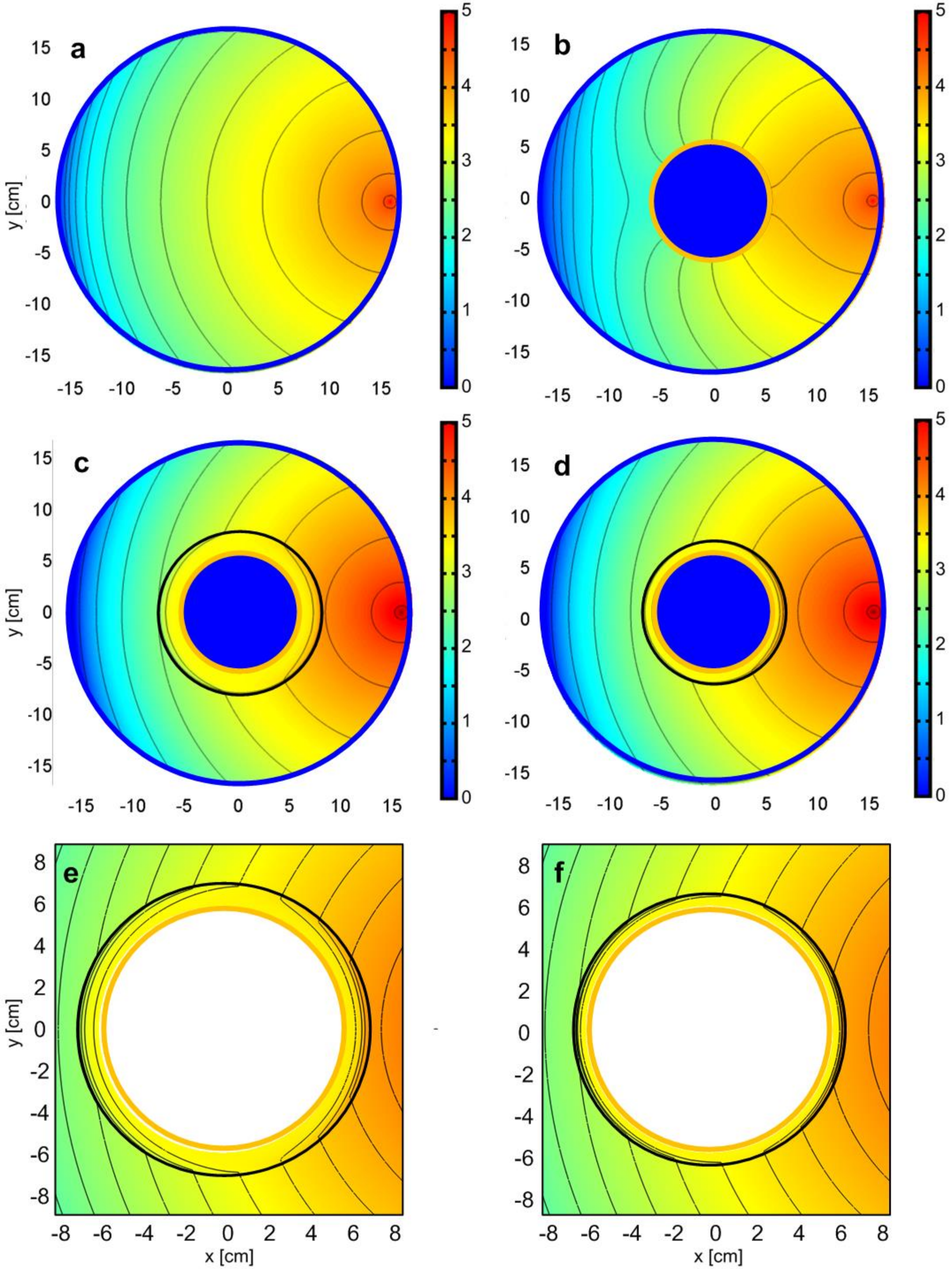
$$\xi' = (\xi_2 - \xi_1)\xi / \xi_2, \quad \eta' = \eta$$

$$\sigma_\xi ' = \frac{\xi_2 - \xi_1}{\xi_2} \sigma_0$$

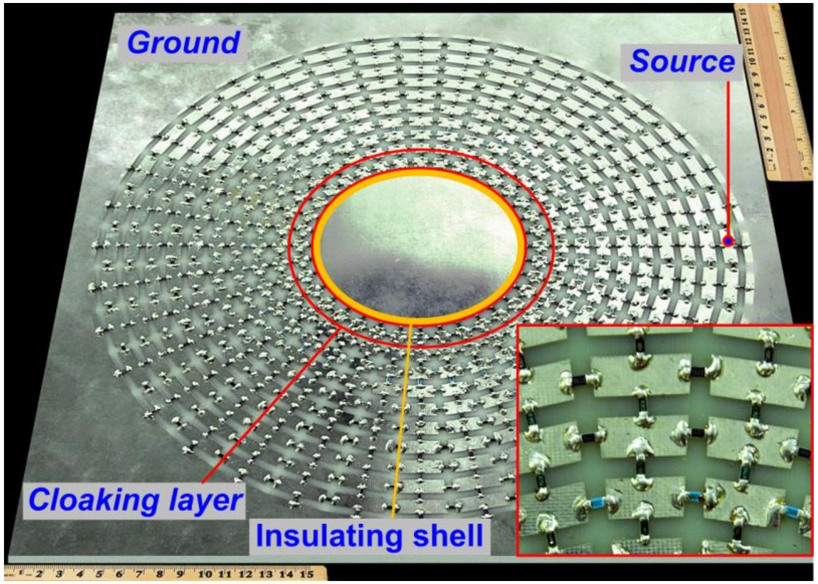
$$\sigma_\eta ' = \frac{\xi_2}{\xi_2 - \xi_1} \sigma_0$$

Homogeneous and Anisotropic; Perfect cloaking effect

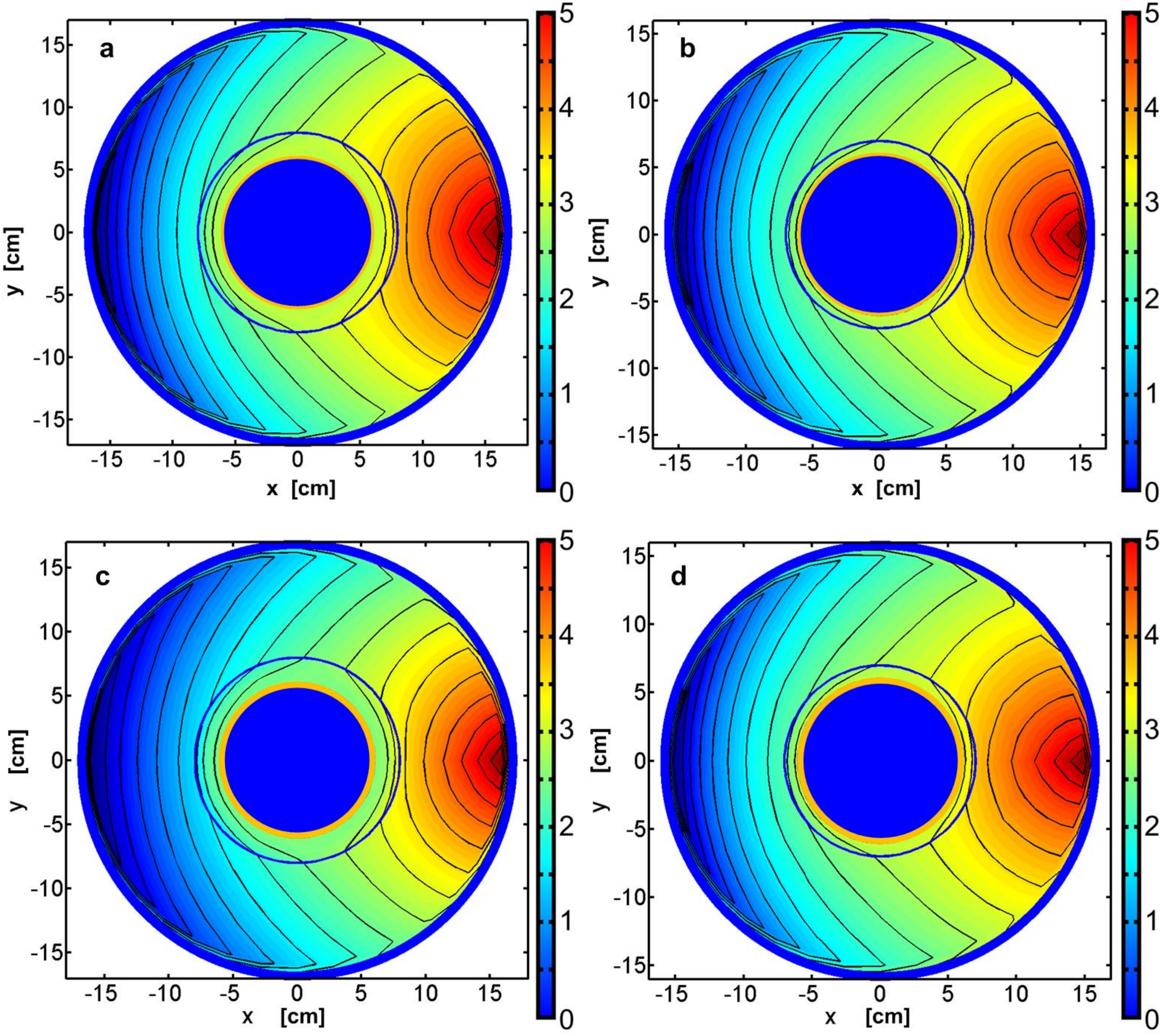
Jiang et al., *APL* 102, 014102 (2013)



Ultrathin but Nearly Perfect!



Ground
Source
Cloaking layer
Insulating shell


a
b
c
d
y [cm]
x [cm]
15
10
5
0
-5
-10
-15
-15
-10
-5
0
5
10
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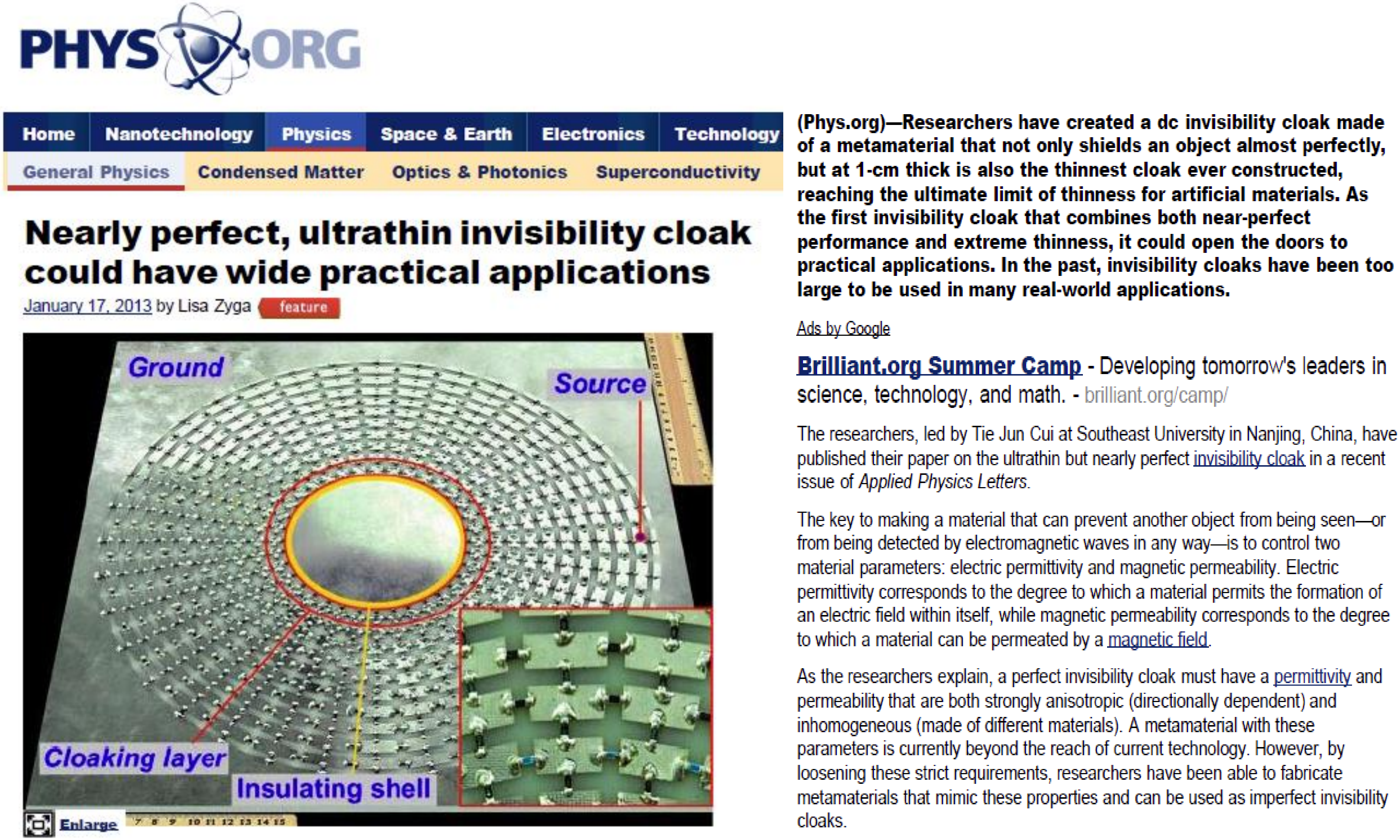
5
4
3
2
1
0

Featured by PHYS ORG



Nearly perfect, ultrathin invisibility cloak could have wide practical applications

January 17, 2013 by Lisa Zyga feature

(Phys.org)—Researchers have created a dc invisibility cloak made of a metamaterial that not only shields an object almost perfectly, but at 1-cm thick is also the thinnest cloak ever constructed, reaching the ultimate limit of thinness for artificial materials. As the first invisibility cloak that combines both near-perfect performance and extreme thinness, it could open the doors to practical applications. In the past, invisibility cloaks have been too large to be used in many real-world applications.

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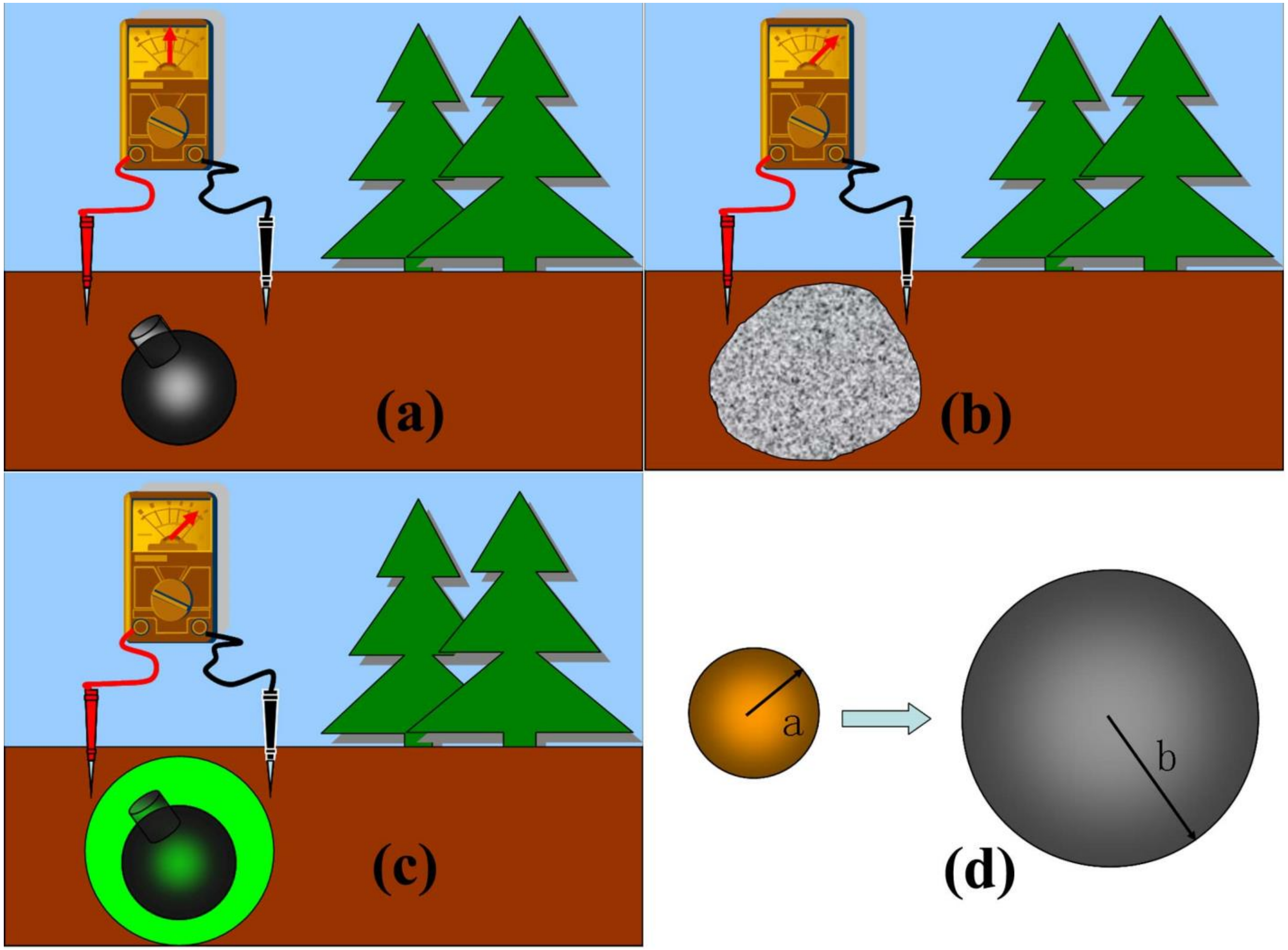
The researchers, led by Tie Jun Cui at Southeast University in Nanjing, China, have published their paper on the ultrathin but nearly perfect invisibility cloak in a recent issue of *Applied Physics Letters*.

The key to making a material that can prevent another object from being seen—or from being detected by electromagnetic waves in any way—is to control two material parameters: electric permittivity and magnetic permeability. Electric permittivity corresponds to the degree to which a material permits the formation of an electric field within itself, while magnetic permeability corresponds to the degree to which a material can be permeated by a magnetic field.

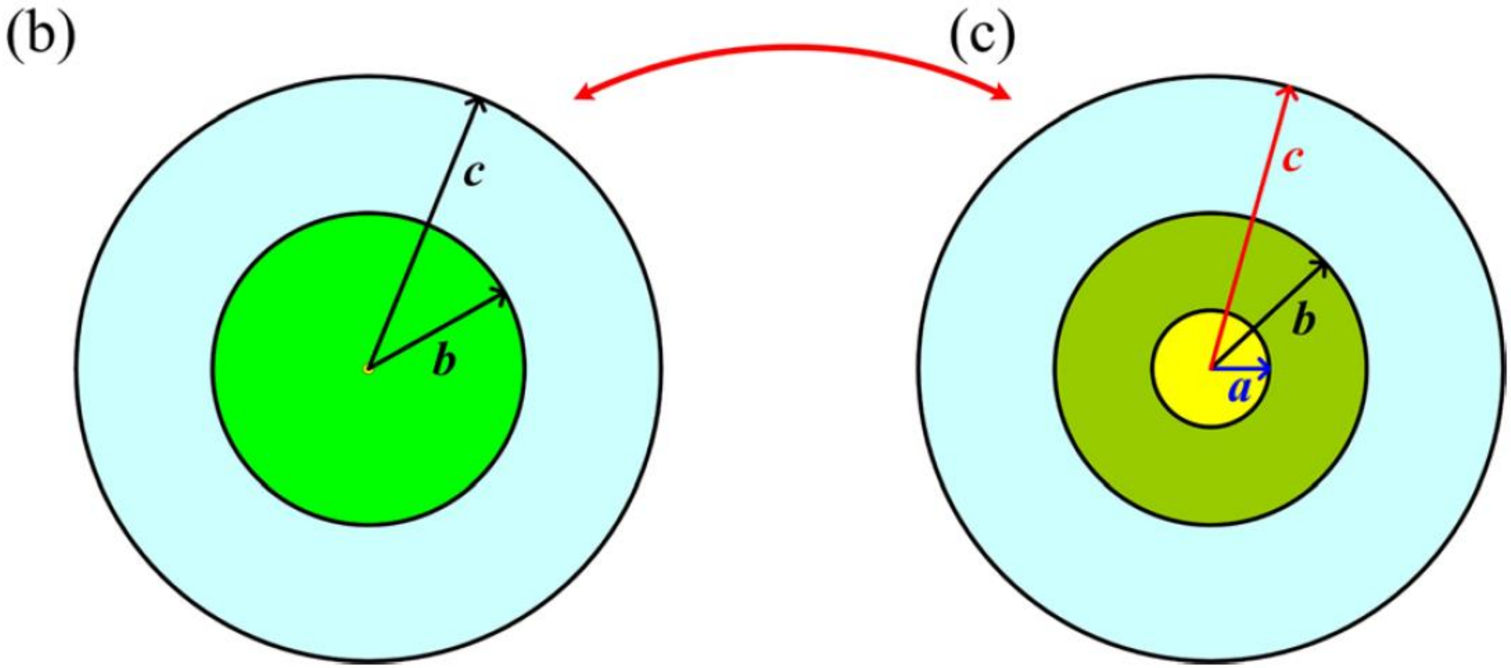
As the researchers explain, a perfect invisibility cloak must have a permittivity and permeability that are both strongly anisotropic (directionally dependent) and inhomogeneous (made of different materials). A metamaterial with these parameters is currently beyond the reach of current technology. However, by loosening these strict requirements, researchers have been able to fabricate metamaterials that mimic these properties and can be used as imperfect invisibility cloaks.

dc Illusion Device



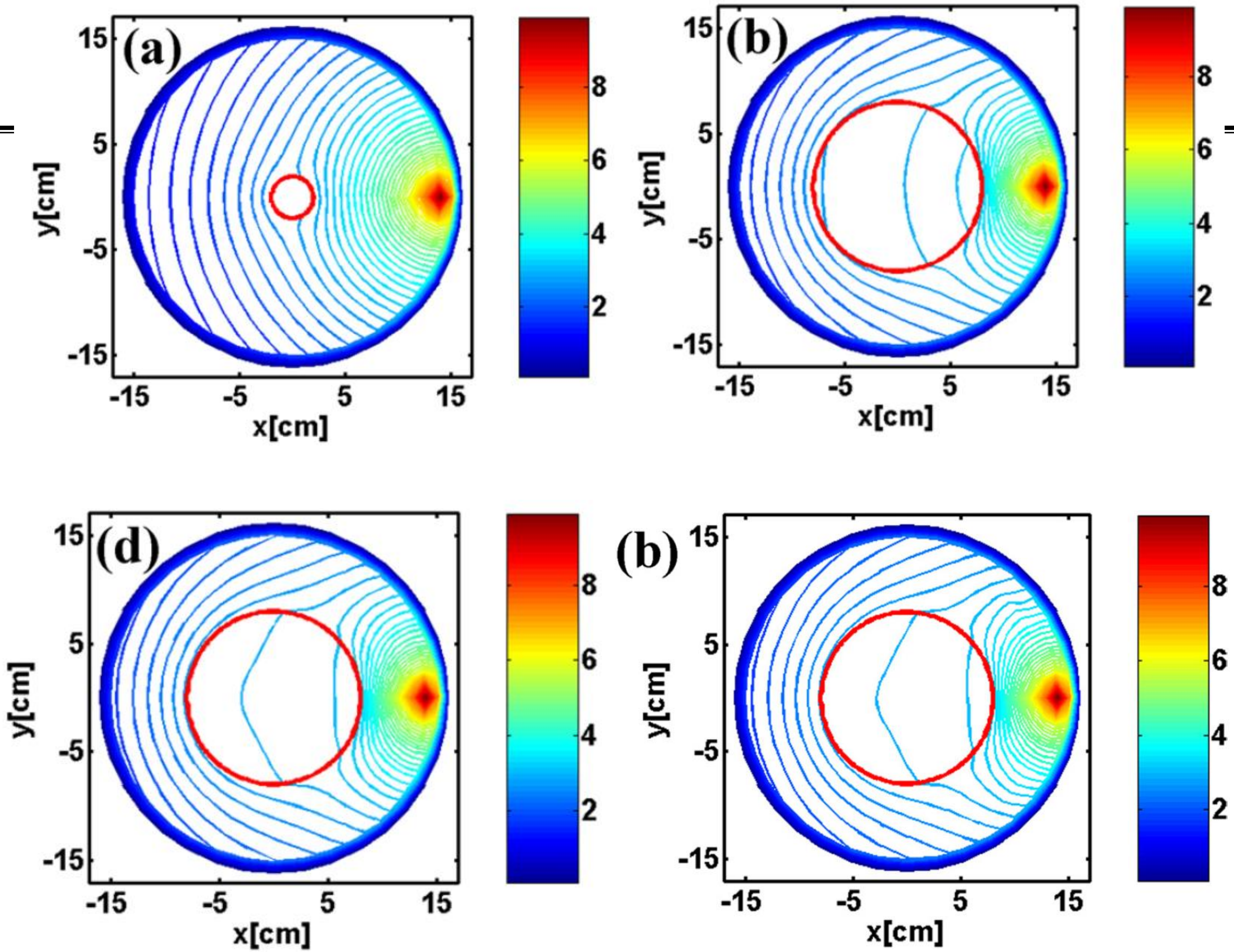
Liu et al., APL 101, 051905 (2012).

dc Illusion Device

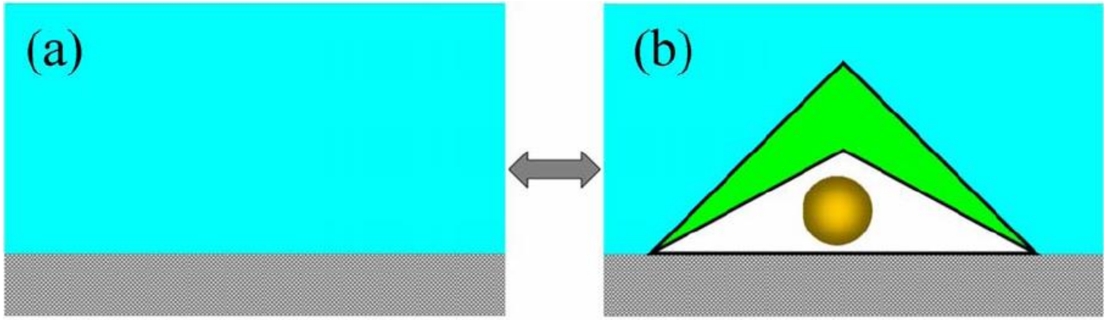


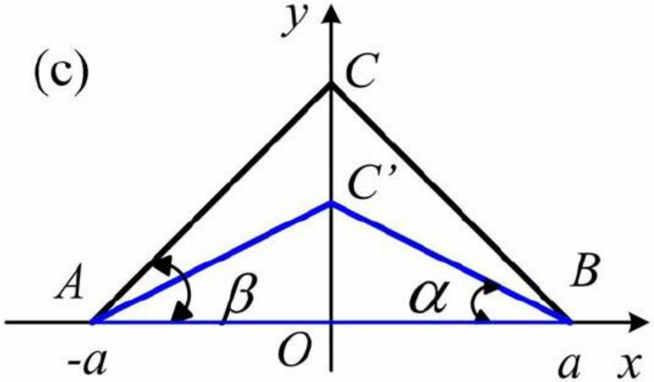
$$\rho' = \frac{b-a}{b}\rho + a, \quad \varphi' = \varphi, \quad z' = z,$$

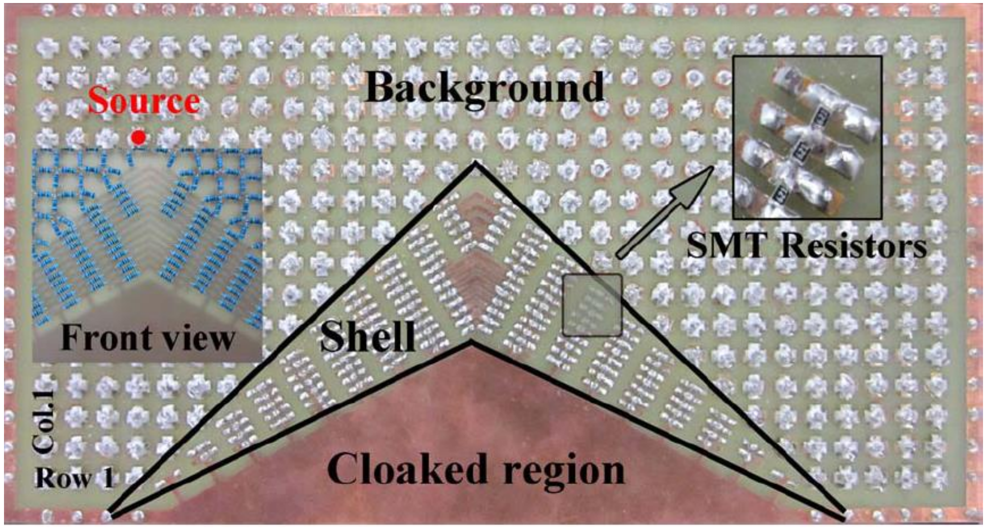
$$\bar{\bar{\sigma}}' = \Lambda\left[\frac{b-a}{b}\frac{\rho}{\rho'}, \frac{b}{b-a}\frac{\rho'}{\rho}, \frac{b}{b-a}\frac{\rho}{\rho'}\right]\sigma.$$


(a)
(b)
(d)
(b)
15
5
-5
-15
y[cm]
-15
-5
5
15
x[cm]
2
4
6
8

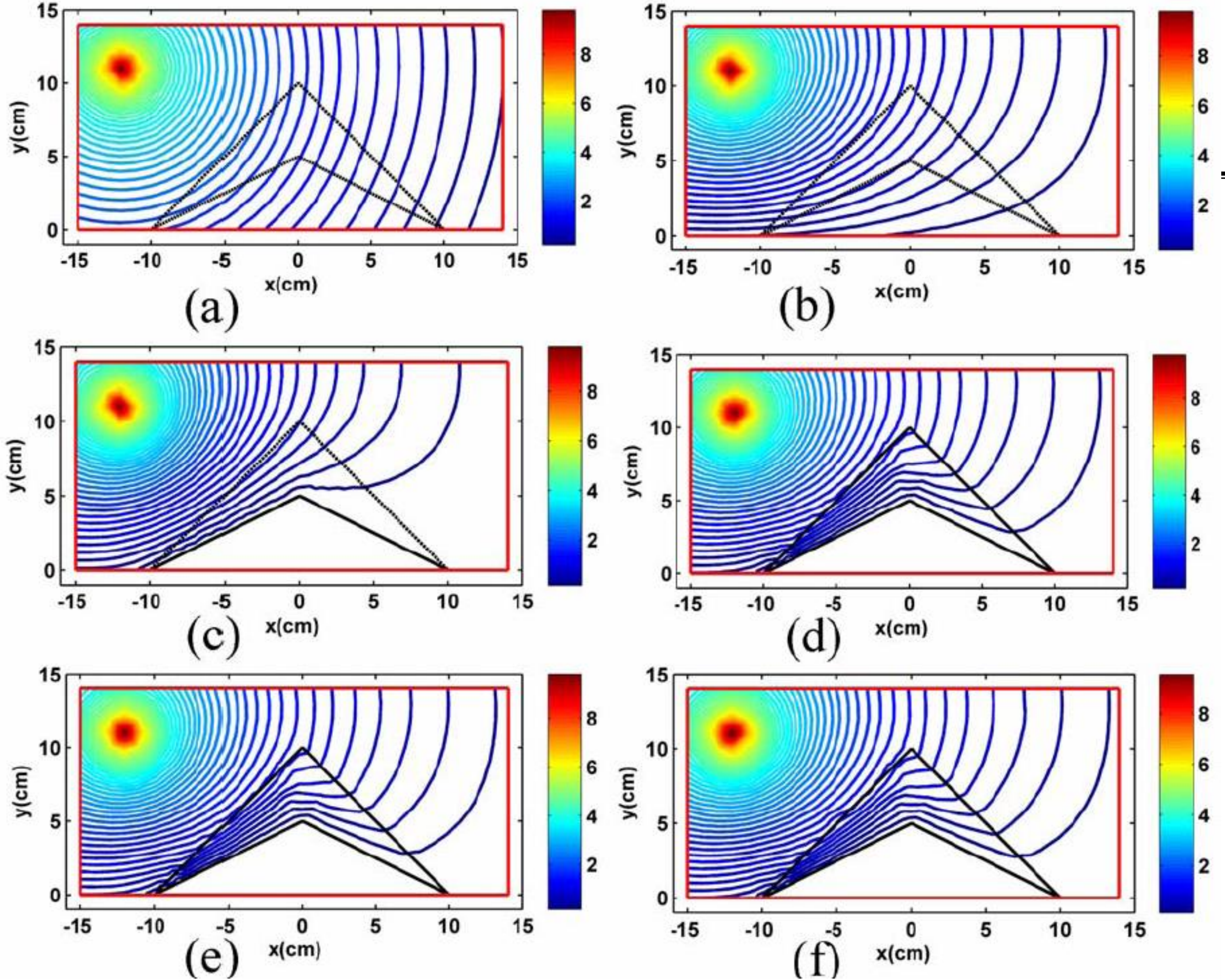
dc Carpet Cloak




(c)
y
C
C'
A
B
β
α
-a
O
a
x

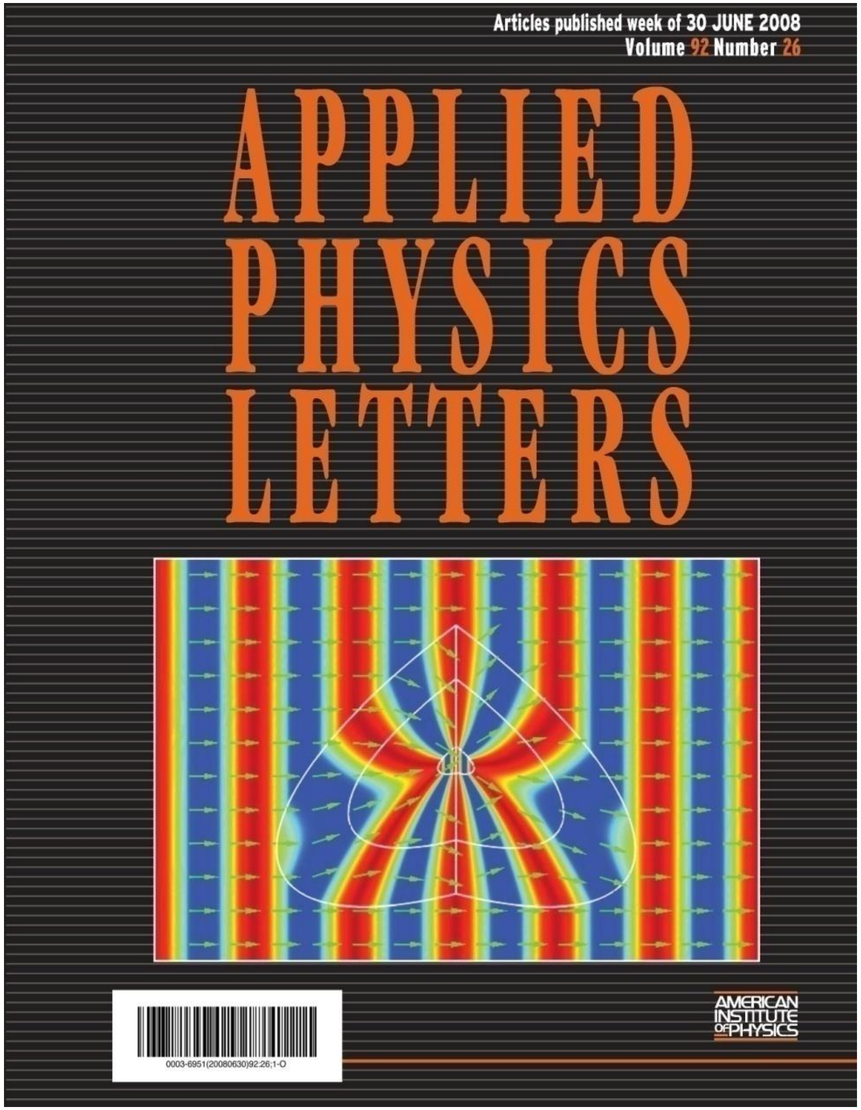

Source
Background
SMT Resistors
Front view
Shell
Col.1
Row 1
Cloaked region

Mei et al., Optics Express 20, 25758-25765 (2012)



(a)
(b)
(c)
(d)
(e)
(f)
x(cm)
y(cm)
-15
-10
-5
0
5
10
15
2
4
6
8

Arbitrarily-Shaped Concentrator

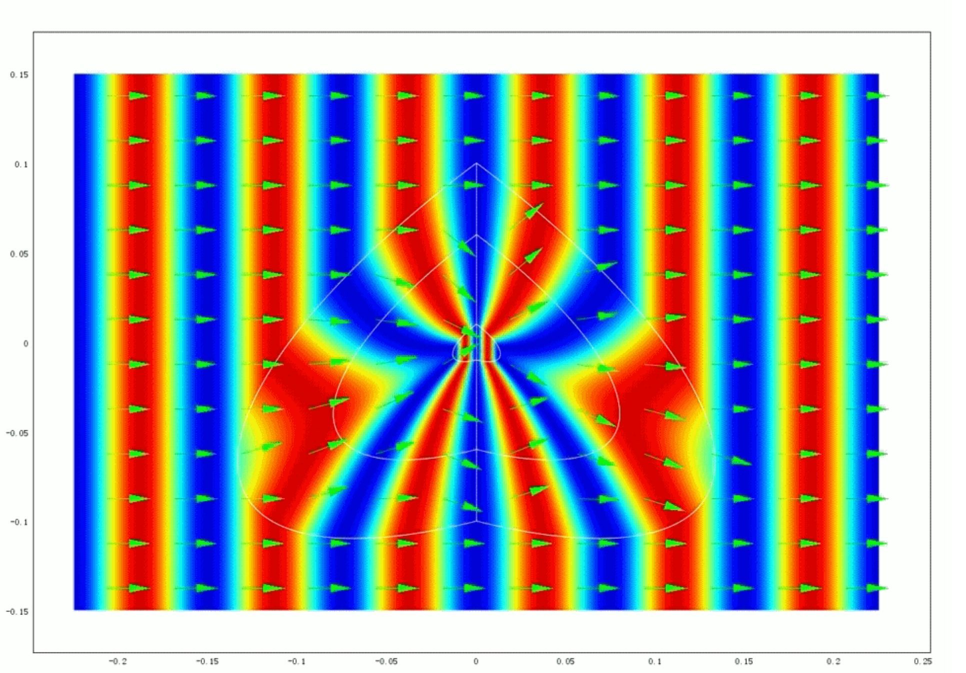


Articles published week of 30 JUNE 2008
Volume 92 Number 26

APPLIED PHYSICS LETTERS

AMERICAN INSTITUTE OF PHYSICS

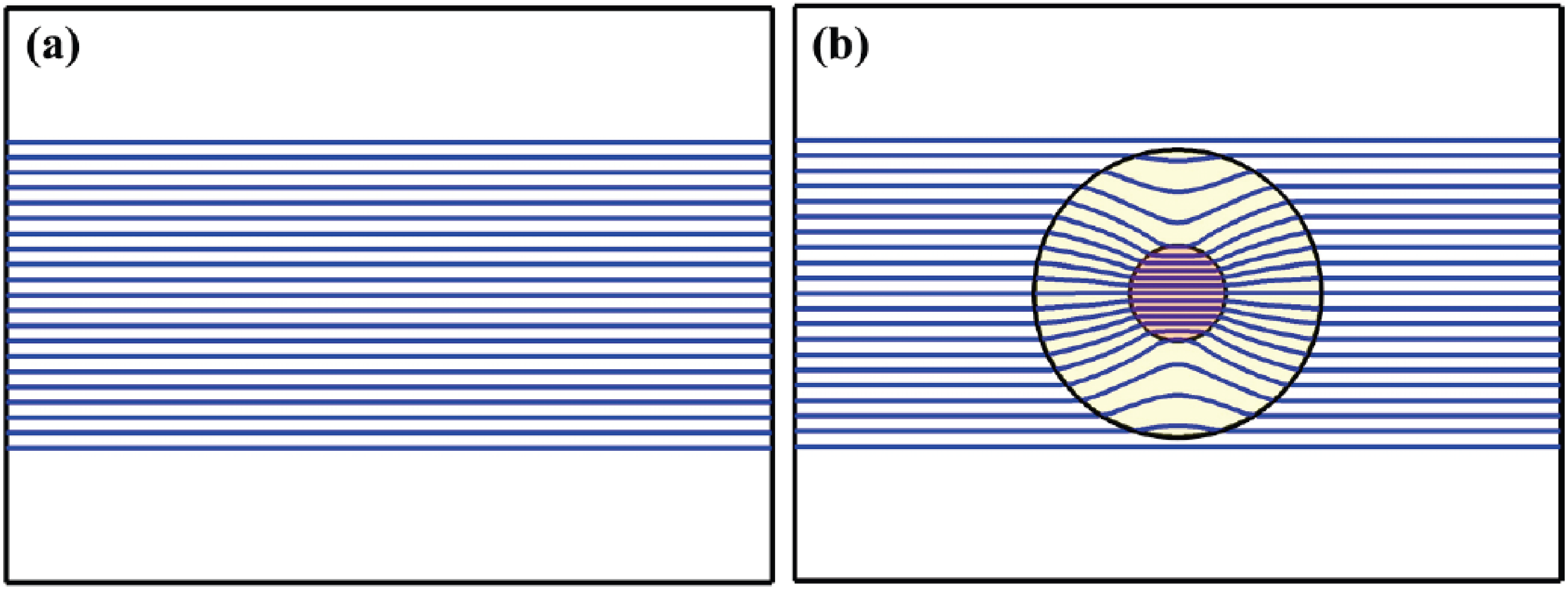
0003-6951(20080630)92:26;1-O



- **The rectangular concentrator can be easily used in amplifying plane waves.**

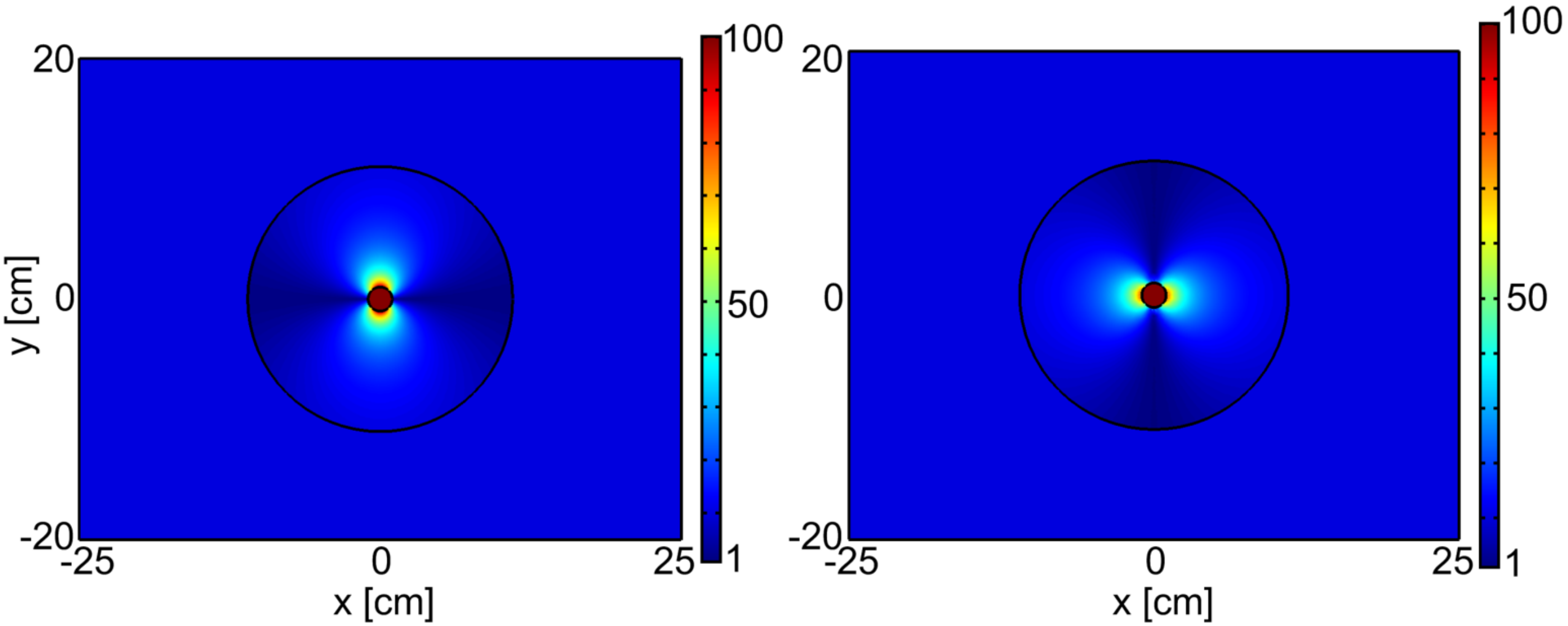
Jiang, et al., *APL* 92, 264101 (2008)

dc Concentrator

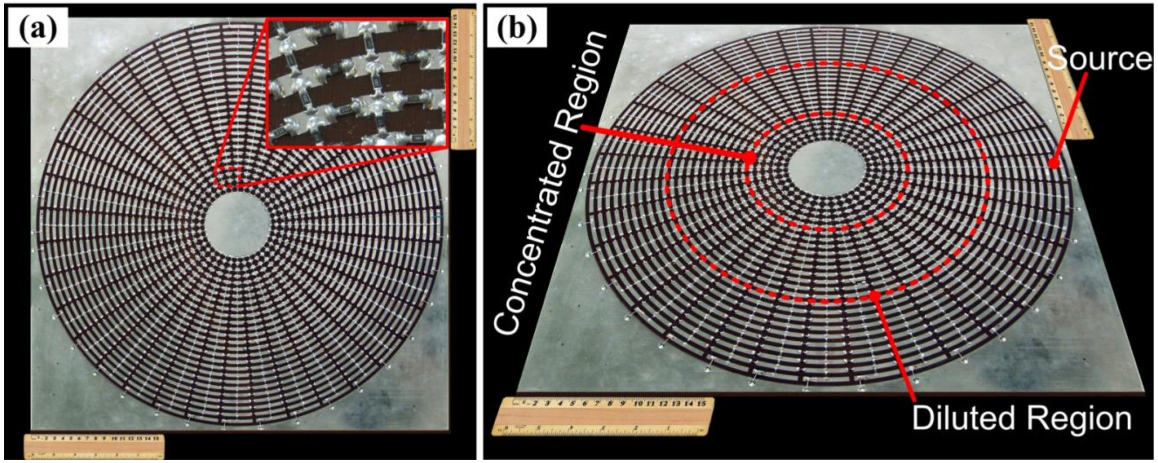


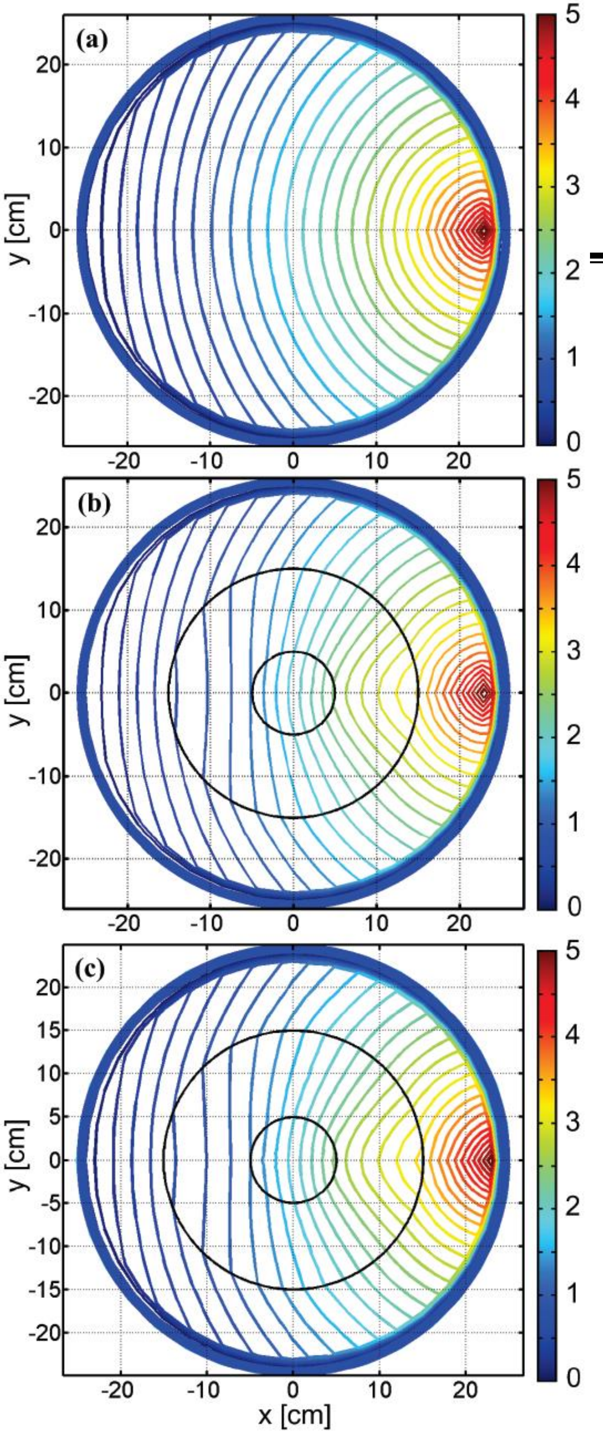
Jiang, Luo, and Cui, *Scientific Reports*, vol. 2, 956 (2012)

dc Concentrator



dc Concentrator

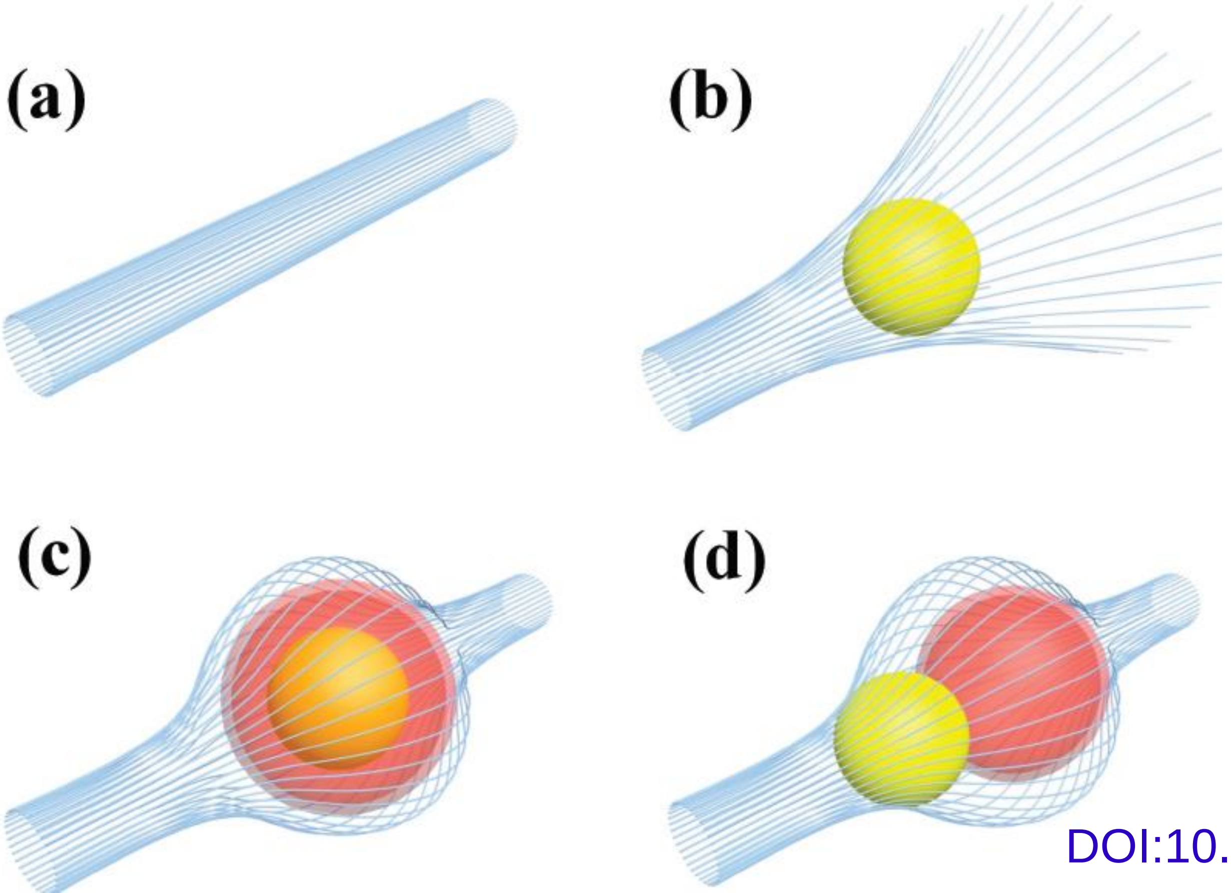

(a)
(b)
Concentrated Region
Source
Diluted Region



dc Exterior Electric Cloak

Lai et al., *PRL* 102, 093901, **2009**

Exterior Cloak: Folding-operation transformation; Only theoretical work

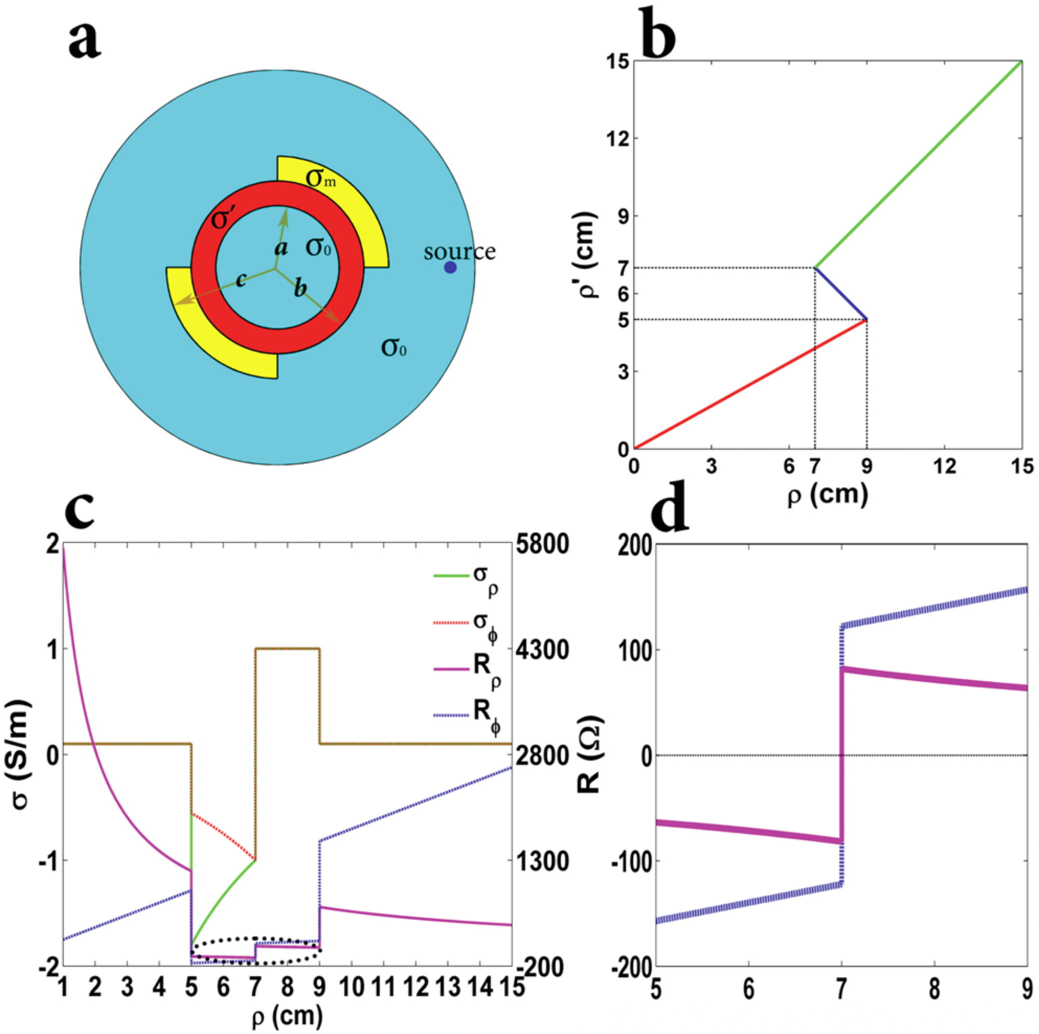


We propose dc exterior cloak, and make realization using active devices.

Yang et al., Adv. Fun. Mat., DOI:10.1002/adfm.201300226, 2013

dc Exterior Electric Cloak

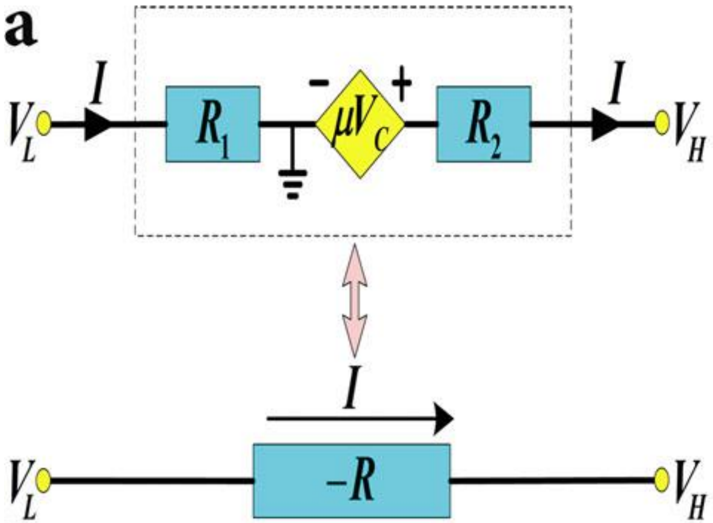
Folding-Operation Transformation



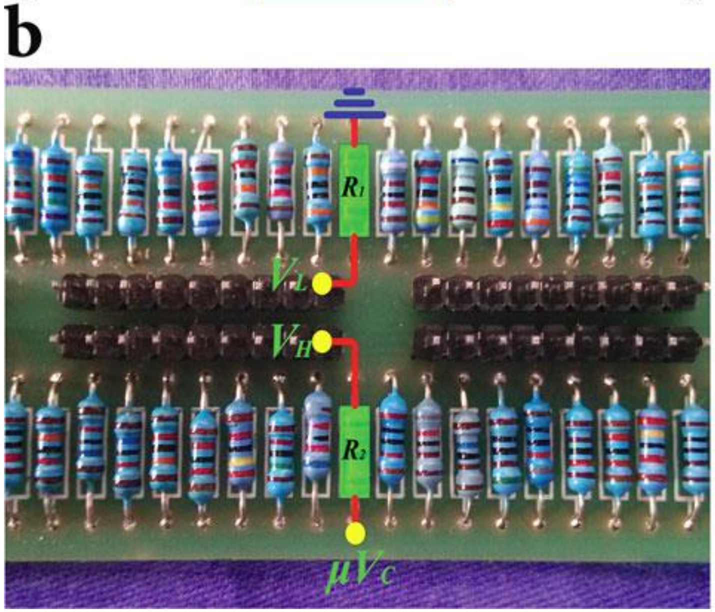
The transformation results in negative conductivity profile in the dc exterior cloak, which can be realized by negative resistors.

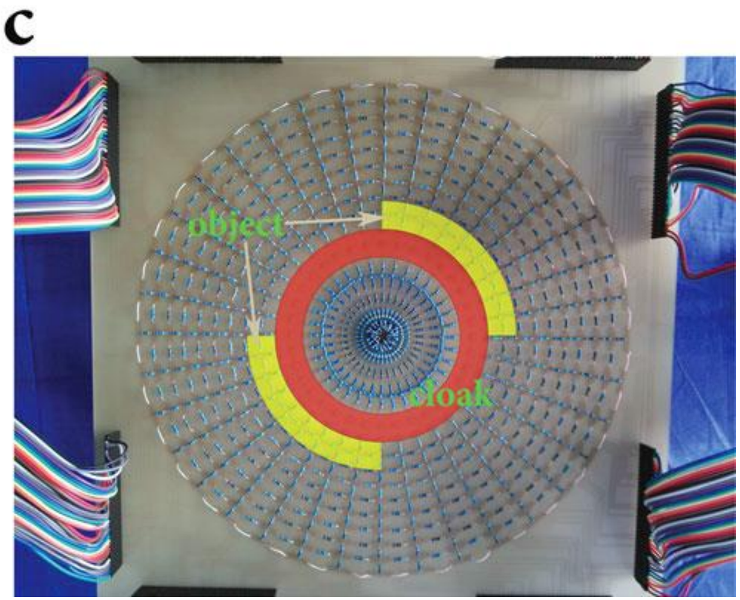
dc Exterior Electric Cloak

F. Yang et al., Adv. Fun. Mat.,
DOI:10.1002/adfm.201300226, 2013

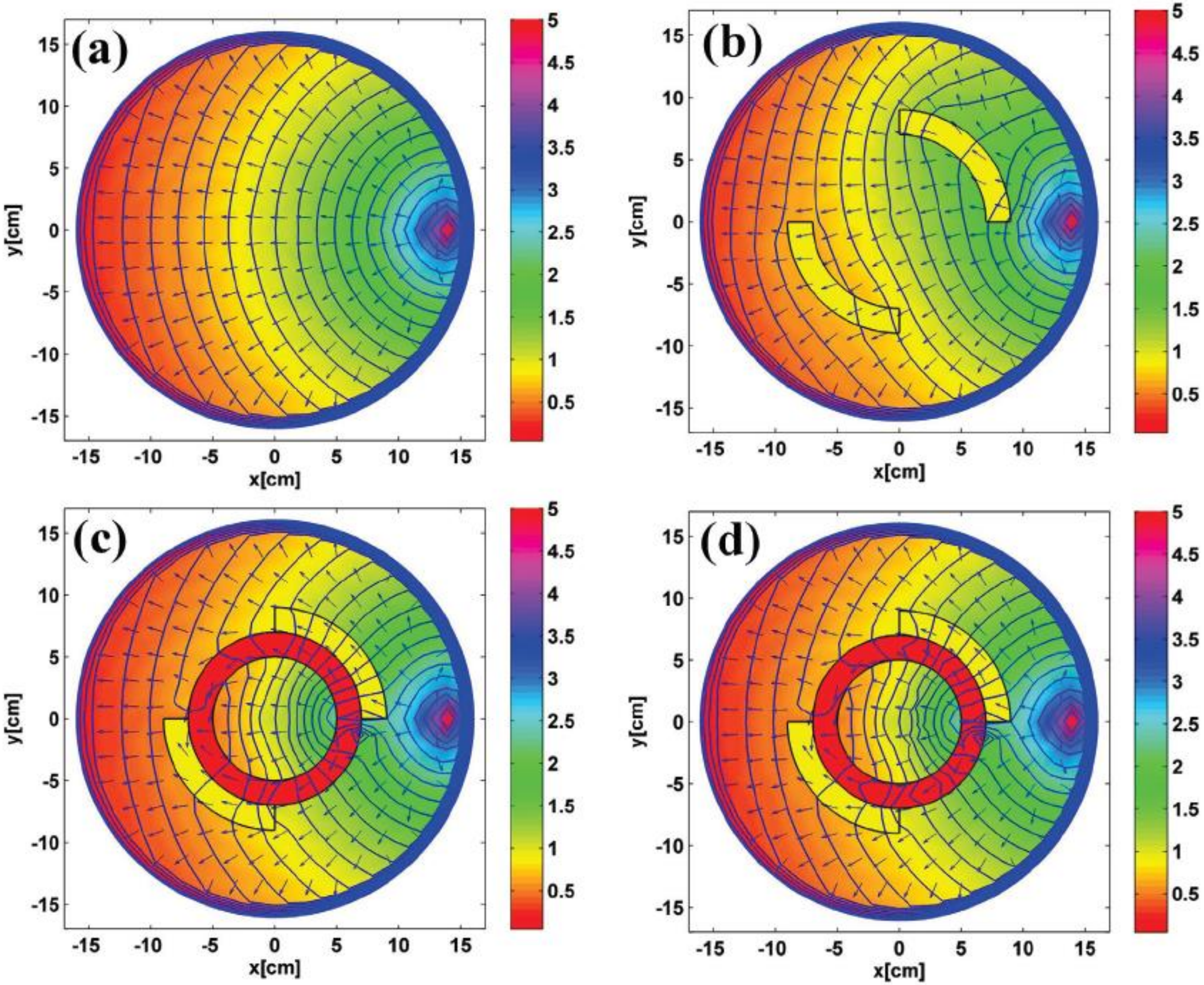

a
V_L
I
R_1
-
μV_C
+
R_2
I
V_H
I
V_L
-R
V_H

Realization of dc Exterior Cloak


b
R1
V_L
V_H
R2
μV_C


c
object
cloak

dc Exterior Electric Cloak



Active dc Electric Cloak

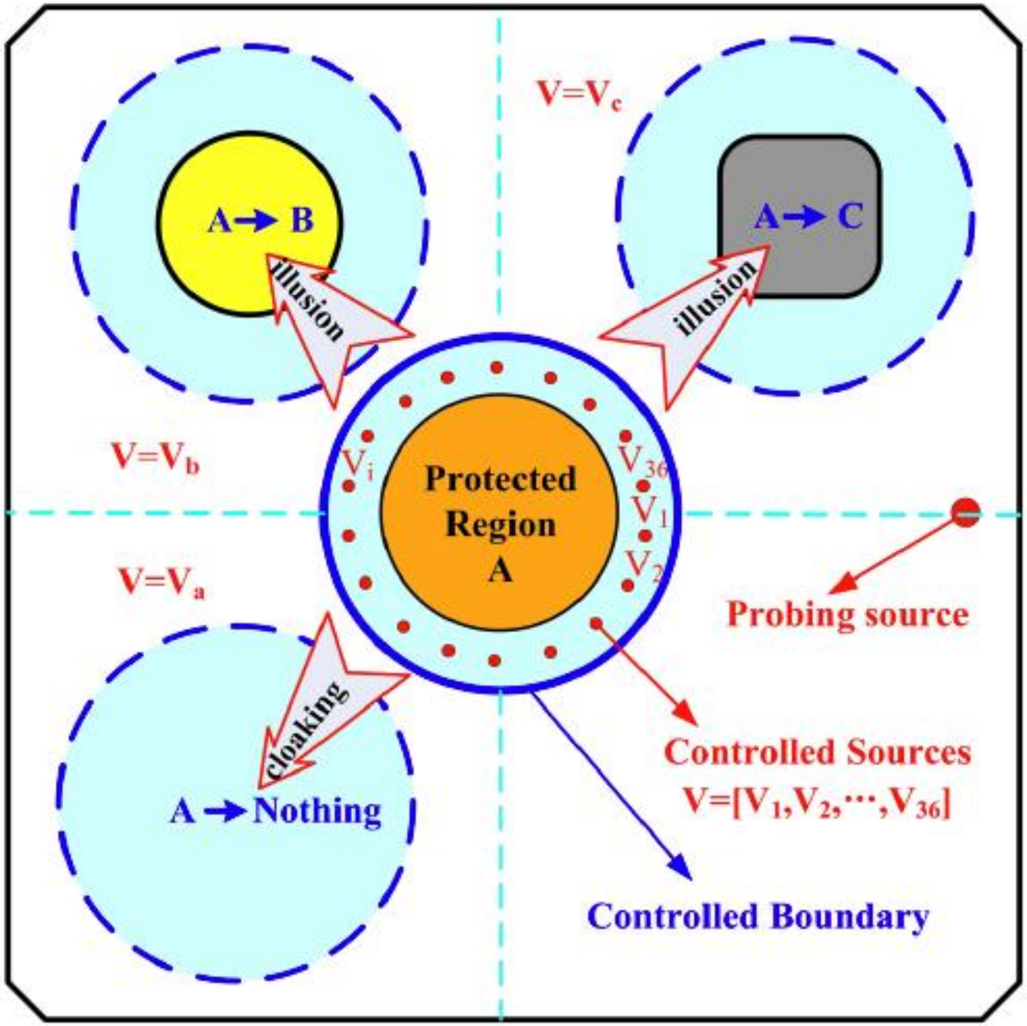
PRL **111,** 173901 (2013) PHYSICAL REVIEW LETTERS week ending 25 OCTOBER 2013

Experiments on Active Cloaking and Illusion for Laplace Equation

We present the first experiment on active cloaking and propose an active illusion for the Laplace equation. We surround the central region with controlled sources to protect it from outside detection. We show that by dynamically changing the controlled sources, the protected region can be cloaked or disguised as different objects (illusion).

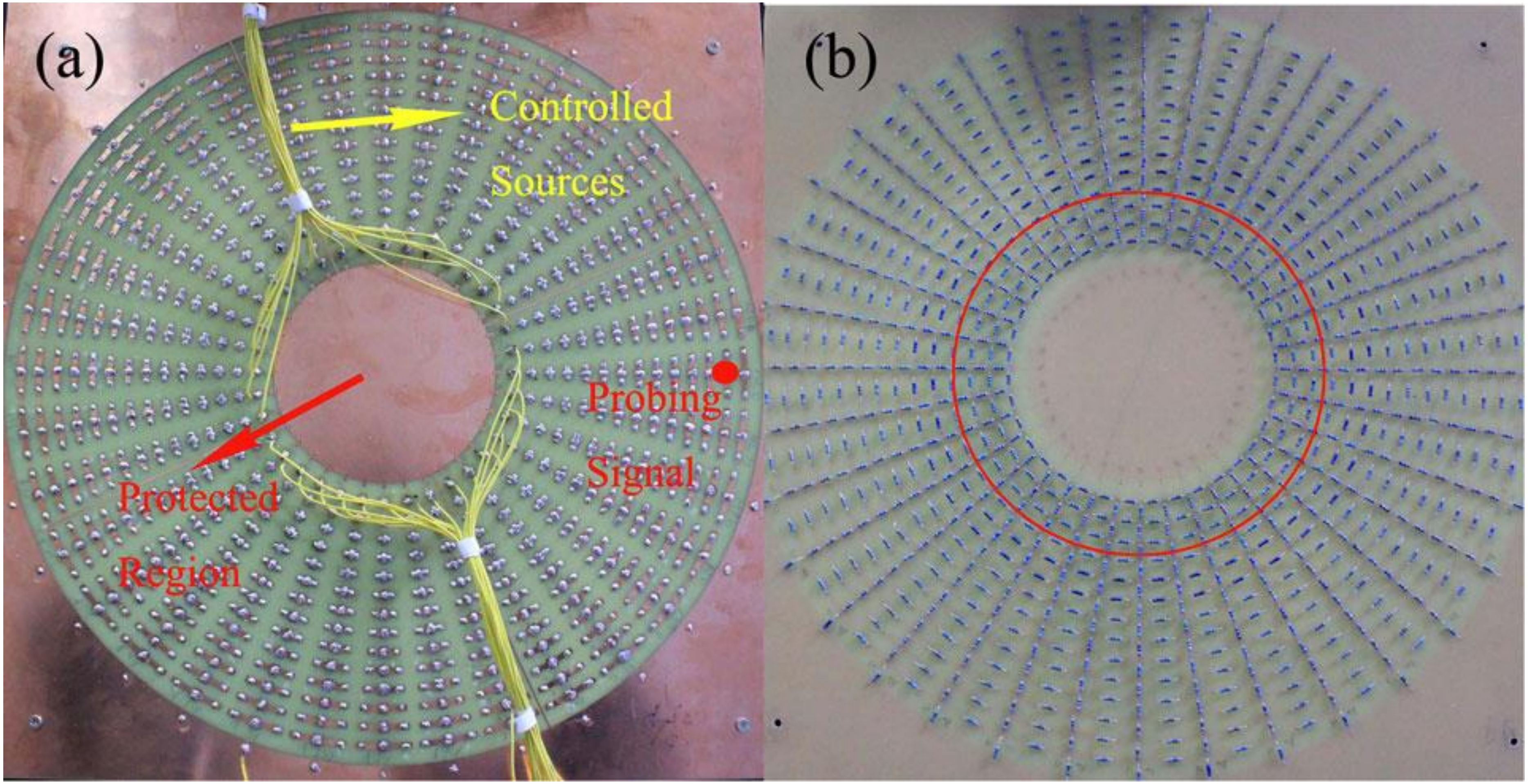
Ma et al., *PRL* 111, 173901, **2013**

Active dc Electric Cloak


V=V$_c$
A → B
A → C
illusion
illusion
V=V$_b$
V$_i$
Protected Region A
V$_{36}$
V$_1$
V$_2$
V=V$_a$
Probing source
cloaking
A → Nothing
Controlled Sources
V=[V$_1$,V$_2$,⋯,V$_{36}$]
Controlled Boundary

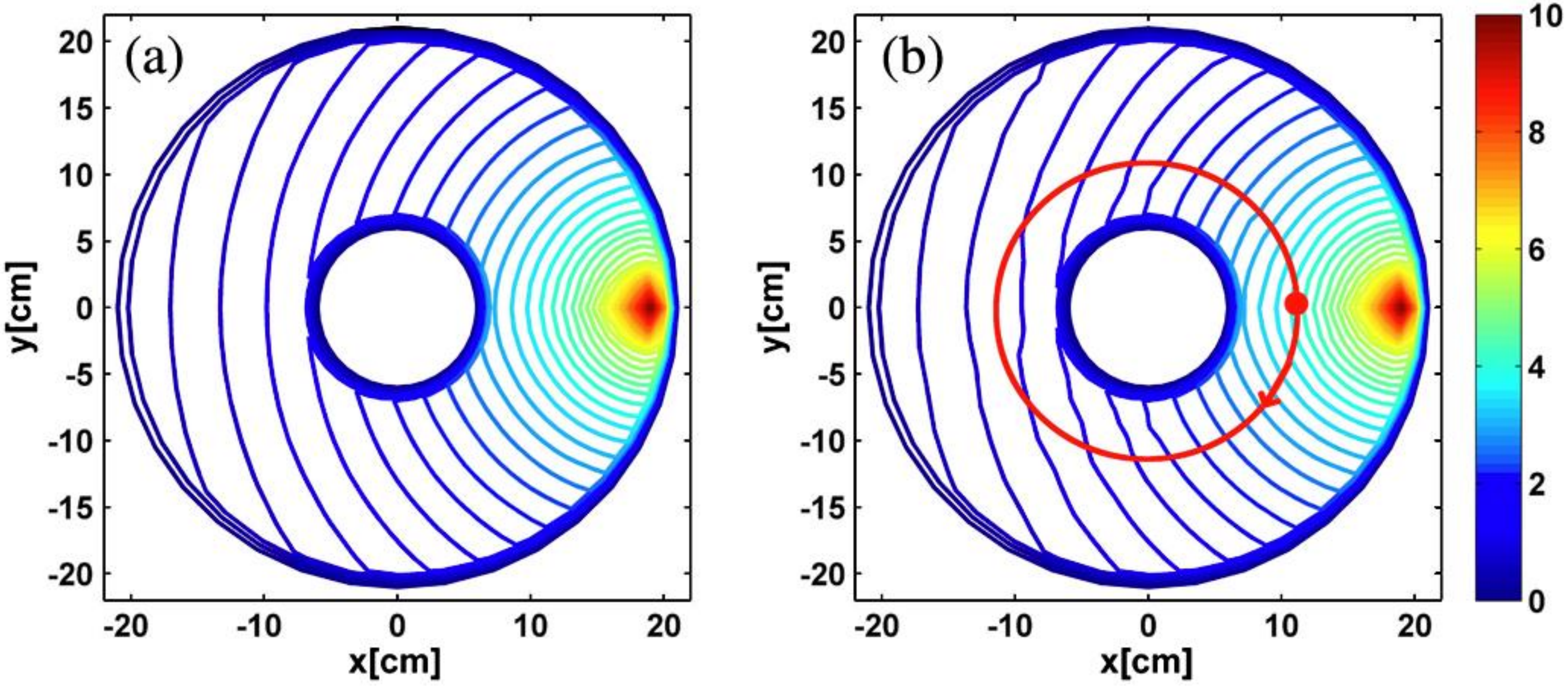
Ma et al., *PRL* 111, 173901, **2013**

Active dc Electric Cloak


(a)
Controlled
Sources
Probing
Signal
Protected
Region
(b)

Ma et al., *PRL* 111, 173901, **2013**

Active dc Electric Cloak



The simulated (left) and measured (right) potential distributions for the cloaking case.

Summary

- We propose a cheaper but efficient way to represent transformation optics in the dc limit.
- All the TO devices proposed in theory in time-varying fields can be realized in their counterparts in the dc field using the resistor network.
- The method can be extended to control the heat transfers, which is also governed by the Laplace equation.

Thank you!

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